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EFFECTIVE HEAT AND MOISTURE TRANSPORT PROPERTIES OF SPRUCE

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Abstract: *The present paper is devoted to the determination of effective properties of wood needed in the macroscopic description of heat and moisture transport in the framework of multi-scale computational scheme. Although computationally feasible, we do not attempt to account for all microstructural details and instead adopt a simplified analytical micromechanical model based on the Mori-Tanaka homogenization scheme to derive effective thermal conductivities of wood and effective diffusivities for the bound water present in the cell wall of wood and water vapor transported through lumens. As one particular example we consider a spruce wood as representative of softwood and most common in the production of glued laminated timber beams.*

Keywords: Spruce, Thermal conductivity, Moisture diffusivity, Homogenization, Mori-Tanaka method.

1. Introduction

An important issue in the safety and reliability design of timber structures is the prediction of the transport of heat and moisture in various structural elements and their impact on both material and structural durability, strength and stiffness. In this regard, the moisture induced stresses in particular belong to principal contributors triggering the evolution of failure. Owing to a considerable complexity in both the material and geometrical description the computational modeling typically relies on hierarchical or staggered solution where the mechanical and transport analyses are decoupled. This approach is also exploited in our current research effort.

The present contribution addresses the mass transport description with emphases on the evolution of moisture content being the stepping stone for the mechanical analysis. It has been suggested, see e.g. (Krabbenhopft et al., 2004 and Frandsen et al., 2007), to abandon a traditional description of one-phase flow in the modeling of transient moisture transport in wood and consider a two-phase flow of bound water in cell walls and water vapor in lumens in the framework of so called non-Fickian moisture transfer. This approach has also been examined in a series of papers by (Eitelberger et al., 2011c, 2011d). Therein, a special procedure for the evaluation of the coupling term, the sorption rate, based on homogenization has been introduced. As also indicated in (Eitelberger et al., 2011c) the two moisture diffusion equations have to be supplemented by the energy conservation equation to account for the changes in energy caused by adsorption and desorption between the water vapor and the bound water and the energy transport owing to the diffusion of both water phases. The complete set of equations to be solved on macroscale is presented in Fig. 1.

Clearly, the solution of macroscopic problem 1 requires the knowledge of effective transport properties such as the macroscopic diffusivities of bound water (D_b) and water vapor (D_v), and the effective thermal conductivity λ. Since these depend on the current value of moisture content and temperature, the solution of the two problems has to be carried out adopting a coupled multiscale computational strategy. The need

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for efficiency usually calls for the application of analytical homogenization schemes such the Mori-Tanaka method discussed next, see e.g. Benveniste (1987) and (Šejnoha et al., 2013).

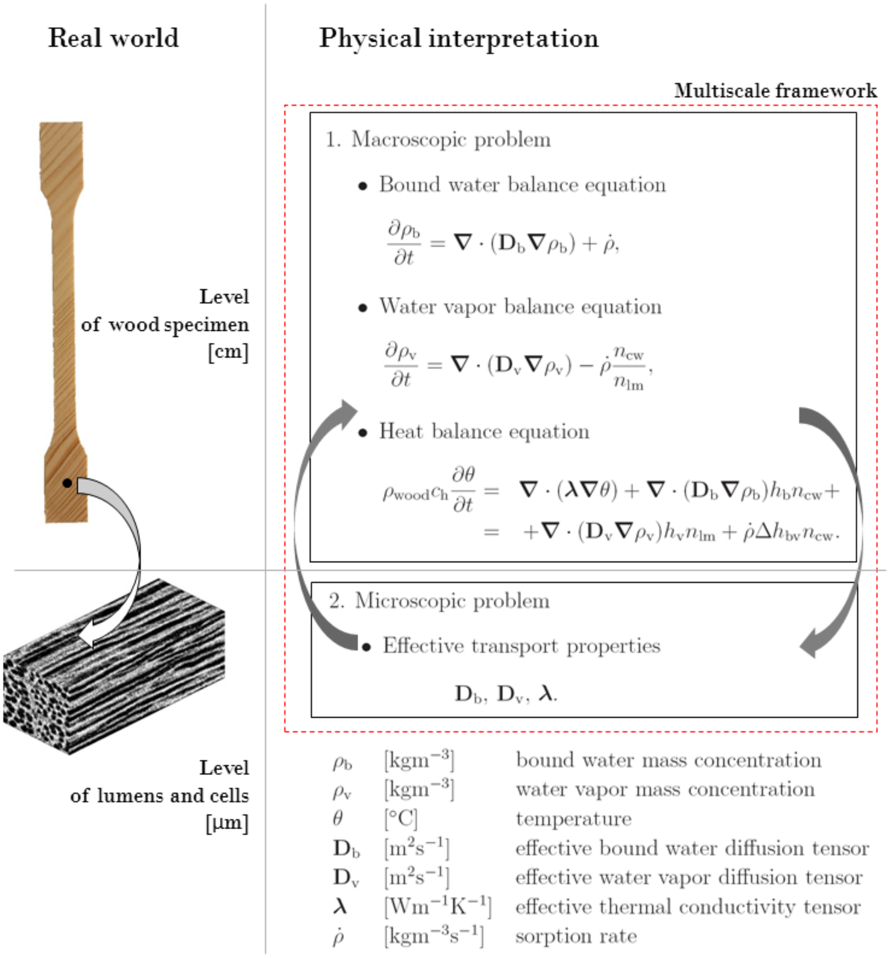

Fig. 1: Summary of material parameters and governing equations to describe the coupled heat and moisture transport in wood.

2. Determination of effective properties

The theoretical background for the derivation of effective transport properties of wood is well described in (Eitelberger et al., 2011a, 2011b). Henceforth, we summarize only the most essential steps with particular application to spruce wood used in the study of glued laminated timber beams.

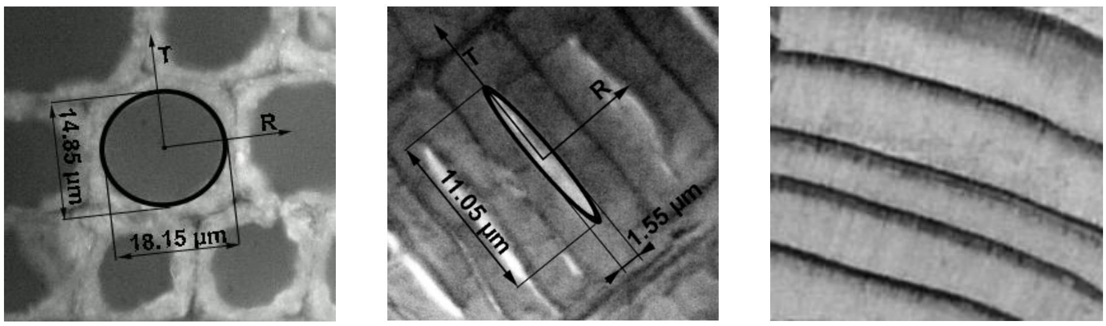

Fig. 2: Microstructure of wood: earlywood, latewood, early and late-wood transition.

2.1. Evaluation of effective properties

Limiting our attention to moisture transport we start with local constitutive equations in the form

$$\mathbf{J}^{\rho}_{cwm} = -\mathbf{D}_{cwm} \nabla \rho_{cwm} = -\widehat{\mathbf{D}}_{cwm} \nabla \rho \tag{1}$$

$$\mathbf{J}^{\rho}_{air} = -\mathbf{D}_{air} \nabla \rho_{air} = -\widehat{\mathbf{D}}_{air} \nabla \rho \tag{2}$$

where $\mathbf{D}_{cwm}$ and $\mathbf{D}_{air}$ are the diffusivity tensors of cell wall and air, respectively. In (Eitelberger et al., 2011a) these tensors were reformulated ($\widehat{\mathbf{D}}_{cwm}, \widehat{\mathbf{D}}_{air}$) to relate the local phase fluxes and the gradient of macroscopic concentration. The simple volume averaging then reads

$$\mathbf{D}_{AV}^{hom} = c_{cwm}\widehat{\mathbf{D}}_{cwm} + c_{air}\widehat{\mathbf{D}}_{air} \quad (3)$$

In the spirit of the Mori-Tanaka method the effective diffusivity tensor is provided by

$$\mathbf{D}_{MT}^{hom} = c_{cwm}\mathbf{D}_{cwm}\mathbf{A}_{cwm} + c_{air}\mathbf{D}_{air}\mathbf{A}_{air} \quad (4)$$

where $\mathbf{A}_{cwm}$ and $\mathbf{A}_{air}$ are the Mori-Tanaka localization tensors relating the macroscopic and phase concentration gradients as

$$\nabla\rho_{cwm} = \mathbf{A}_{cwm}\nabla\rho, \quad \nabla\rho_{air} = \mathbf{A}_{air}\nabla\rho \quad (5)$$

These tensors, see e.g. (Šejnoha et al., 2013), depend on the properties of cell walls and lumens and the shape and orientation of lumens. The latter differs for the early and latewood as evident from Fig. 2. The volume fractions c_{cwm}, c_{air} were found from image analysis of actual microstructures also depicted in Fig. 2. The last homogenization step to get the properties of wood at the level of annual rings reduces to the application of simple Voight and Reuss bounds as

$$\mathbf{D}_{AR} = c_{EW}\mathbf{D}_{EW} + c_{LW}\mathbf{D}_{LW}, \quad \mathbf{D}_{AR}^{-1} = c_{EW}\mathbf{D}_{EW}^{-1} + c_{LW}\mathbf{D}_{LW}^{-1} \quad (6)$$

where c_{EW}, c_{LW} are the volume fractions of earlywood and latewood, respectively, found again from images such as the one in Fig. 2c. Similar procedure leads to the effective thermal conductivity.

2.2. Results

As an example, we provide in Tabs. 1 and 2 the results derived for spruce wood used already in the stochastic analysis of glued laminated timber beams where similar type of homogenization was used to get the effective elastic properties, see (Šejnoha et al., 2017). The values correspond to the selected moisture content 10 % and temperature θ = 20 °C.

Tab. 1: Effective thermal conductivities.

Homogenization step	**Phase**	**c_r [-]**	**λ_{11} [W/mK]**	**λ_{22} [W/mK]**	**λ_{33} [W/mK]**
Step 1	Lignin	0.39	0.39	0.39	0.39
3 phases	Hemicellulose	0.37	0.34	0.34	0.34
	Water+ext	0.24	0.60	0.60	0.60
Step 2	Polymer network	0.65	0.42	0.42	0.42
2 phases	Cellulose	0.35	0.26	0.26	1.04
Step 3	Cell wall	0.58/0.82	0.36	0.36	0.64
2 phases	Lumen	0.42/0.18	0.03	0.03	0.03
Step 4	Earlywood	0.67	0.17	0.16	0.38
2 phases	Latewood	0.33	0.18	0.29	0.53
	Annual ring	-	0.17	0.20	0.43

Tab. 2: Effective diffusivities.

Homogenization step	**Phase**	**c_r [-]**	**D_{11} [10^{-12} m^2/s]**		**D_{22} [10^{-12} m^2/s]**		**D_{33} [10^{-12} m^2/s]**	
			AV	M-T	AV	M-T	AV	M-T
Step 1	Cell wall	0.58/0.82	2.26		2.26		5.65	
2 phases	Lumen	0.42/0.18	$26\text{x}10^{-6}$		$26\text{x}10^{-6}$		$26\text{x}10^{-6}$	
Step 2	Earlywood	0.67	6.72	5.90	6.72	5.24	1486.78	2962.00
2 phases	Latewood	0.33	3.36	2.83	3.36	6.30	454.65	11080.00
	Annual ring	-	5.00	4.34	5.01	5.59	960.77	5640.94

While only two step-homogenization scheme is needed when deriving the effective diffusivities, additional two homogenization steps are required on the level of cell walls to get the effective thermal conductivities, see (Eitelberger et al., 2011a, 2011b) from where the phase properties were adopted. The volume fractions of phases below the cell wall level were taken from (Hofstetter et al., 2005). As already mentioned, the volume fractions at the level of lumens and the level of wood rings were derived for the analyzed wood from actual images. Note that at the level of wood the classical Voight and Reuss bounds were exploited, see e.g. (Šejnoha et al., 2013).

3. Conclusions

The present paper reviewed a classical steady state homogenization method based on both the Mori-Tanaka method and simple averaging. The homogenized properties listed in Tabs. 1 and 2 were found for one specific value of the moisture content and temperature. When introduced into the macroscopic transient analysis, recall the macroscopic model 1 in Fig. 1, these have to be calculated at each time integration step.

While the effective thermal conductivities are obtained from the actual step 4 of the homogenization scheme, recall Tab. 1, the diffusivity tensor $\mathbf{D}_b$ follows from Eqs. (3) and (4) when setting the air diffusivities equal to 0. On the contrary, the tensor $\mathbf{D}_v$ is identical to $\mathbf{D}_{air}$, see (Eitelberger et al., 2011c).

Similar types of experiments as presented in (Eitelberger et al., 2012) are currently under way to provide data for the validation of implementation of macroscopic model 1. The realistic prediction of mechanical behavior is highly dependent on the moisture gradients. Therefore, the physically-based model for transport processes in wood presented here profiting from the multi-scale analysis is a crucial step in the safety and reliability design of timber structures.

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