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DETERMINATION OF NON-LINEAR ROLL DAMPING COEFFICIENTS FROM MODEL DECAYTEST

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Abstract: *A conventional method to estimate the damping of an oscillatory system is the execution of amplitude extinction tests. In the specific case of the roll motion of a vessel, the so-called roll decay tests are performed in a model basin. During these tests, the system is posed in an imbalance condition by an external moment. For systems where the damping is far below critical value, the transient decay to equilibrium condition is oscillatory. There are several methodologies to analyze the decay test time-series, all based on the assumption that a pure roll motion has been reproduced. Through this paper an overview of the most commonly used methods is given and a comparison of the results is shown for a single ship model decay test.*

Keywords: Roll motion, Damping coefficient, Decay test.

1. Introduction

To estimate the damping coefficient of the roll motion for a surface vessel, dedicated model tests are carried out in specialized hydrodynamic laboratories. The specific test to be performed is the so-called rolldecay test. When performed according to the ITTC (International Towing Tank Conference) standard, it consists in measuring the amplitude of the roll motion of the ship model as function of time. To do that, a predetermined transversal inclination must be given to the model and then let it to oscillate up to reaching the equilibrium. The procedure should be performed for all the vessel speeds of interest. From the obtained time-series record there are several methods to analyze the data in order to estimate the damping coefficients, based on non-linear dynamic equation of roll motion. In the theory of Ship motions, it is common practice to linearize the dynamic system by decoupling all the motion equations (Blagoveshchensky, 1964). In such a way only the roll equation is considered by neglecting couplings with other motion as sway and yaw. In such a way the mathematical treatment of the problem results simplified especially when non-linear damping has to be analyzed.Introduction of linear sway or yaw coupling, only slightly complicates the problem. The most commonly used methods to evaluate the coefficients are here described and applied to a roll decay record.

2. Roll motion equation

To describe the pure roll behavior of a vessel subjected toexternal active forces, the following single degree of freedom equation can be written:

$$I_\phi \ddot{\phi} + B_\phi(\dot{\phi}) + C_\phi(\phi) = M_\phi \tag{1}$$

where I_ϕ represents the virtual mass moment of inertia along the longitudinal roll axis, B_ϕ is the damping moment, C_ϕ is the restoring moment and M_ϕ represents the external moment due to waves or other external forces. In the specific case of a standard decay test, the external moment M_ϕ is not considered, leading to:

$$I_\phi \ddot{\phi} + B_\phi(\dot{\phi}) + C_\phi(\phi) = 0 \tag{2}$$

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Equation (2) can be expressed with several orders of non-linearity, depending on the modeling of the damping moment B_ϕ and of the restoring moment C_ϕ.

The damping moment B_ϕ of a vessel can be expressed as a power series of $\dot{\phi}$ and $|\dot{\phi}|$ usually up to the third order but, in case of small oscillations, a second order approximation is sufficient; resulting in:

$$B_\phi(\dot{\phi}) = B_{\phi 1}\dot{\phi} + B_{\phi 2}\dot{\phi}|\dot{\phi}| \quad (3)$$

The restoring moment C_ϕ of a vessel is expressed in the following form:

$$C_\phi(\phi) = \Delta \overline{GZ}(\phi) \quad (4)$$

where Δ is the vessel displacement and $\overline{GZ}(\phi)$ is the righting arm of the vessel. In this case the righting arm can be expressed in terms of odd-order polynomials. In case of small amplitude oscillatioms, a linear righting arm can be considered with the following form:

$$C_\phi(\phi) = \Delta \overline{GM}\phi \quad (5)$$

where $\overline{GM}$ is the so-called metacentric height, function of the center of gravity height, of the ship volume and of the transversal inertia of the waterplane area. So, by considering formulation (3) for the damping moment and (5) for the restoring moment, equation (2) can be rewritten as:

$$I_\phi\ddot{\phi} + B_{\phi 1}\dot{\phi} + B_{\phi 2}\dot{\phi}|\dot{\phi}| + \Delta\overline{GM}\phi = 0 \quad (6)$$

Dividing (6) by I_ϕ the motion equation takes the following non-dimensional form:

$$\ddot{\phi} + 2\nu\dot{\phi} + w\dot{\phi}|\dot{\phi}| + n^2\phi = 0 \quad (7)$$

where 2ν is the linear damping coefficient and w is the quadratic damping coefficient.

3. Logarithmic decrement

The most common way to analyse a decay test is to find the logarithmic decrement of the consecutive oscillation in order to figure out the behaviour of the decay process. Once the oscillation peaks have been extracted from the time series, the analysis will proceed with the determination of the successive oscillations decrements. In case that equation (7) is used for the roll modelling, a particular solution can be found for the first swing, supposing that the first peak is positive. In such a case the term $\dot{\phi}|\dot{\phi}|$ can be defined as $\dot{\phi}^2$ and the solution becomes:

$$\phi = \left(\phi_0 - \frac{2}{3}w\phi_0^2\right)e^{-\nu t}\cos\omega t + \phi_0^2 w e^{-2\nu t}\left(\frac{1}{2} + \frac{1}{6}\cos 2\omega t\right) \quad (8)$$

where the frequency of oscillation ω is defined as $\sqrt{n^2 - \nu^2}$. The decrease in amplitude of the first swing and generally of the $i+1^{th}$ swing can be determinedin the following form:

$$\Delta\phi_{i+1} = \phi_i\left(1 - e^{-\nu T_c/2}\right) + \frac{2}{3}w\phi_i^2 e^{-\nu T_c/2}\left(1 + e^{-\nu T_c/2}\right) \quad (9)$$

where T_c is the period of the oscillation and, in case of a vessel, can be approximated by the rolling period without damping $T=2\pi/n$ (Vlasov, 1930). It must be noted that, by using this kind of notation, all the peaks should be considered. When the procedure is applied only to maxima or minima, then the period to be considered in (9) is T_c instead of $T_c/2$. The quantity of the decrement of the single swings can also be directly determined from the decay record, making the difference between the consecutive peaks.

4. Equivalent linear damping

Another way to evaluate the damping coefficients from the decay test is to use the so-called equivalent linear damping representation. This way to represent the data is coming from a theoretical assumption based on the Newton statement, establishing that, for small damped oscillations with a resistance law in quadratic polynomial form, the decay law for amplitudes can be expressed in approximated form as:

$$\Delta\phi = a_1\phi_m + a_2\phi_m^2 \quad (10)$$

where ϕ_m is the average value of the amplitude of the examined swing. Considering that the restoring moment in equation (7) is linear with respect to ϕ, in the instants where the vessel passes through the equilibrium position the ϕ-dependent part of the equation goes to zero. Means that, for these time instants, the roll equation can be represented as:

$$\phi = f(t)\cos\omega t \quad (11)$$

where *f(t)* is a function slowly decaying with time. If *f(t)* is considered as the envelope curve of the decay oscillation (Pavlenko, 1947), then a relation can be found between linear and quadratic damping, equating the integral of the resistance laws between 0 and ϕ_m. Resulting in:

$$2N = \frac{8}{3\pi} W\omega\phi_m \quad (12)$$

where *2N* and *W* are the damping coefficients in dimensional form. The dimensional linear damping coefficient can be obtained directly from the decay record at each ϕ_m with the following formulation:

$$2N = \frac{2T}{\pi} k\phi_m \frac{1}{\phi_i - \phi_m} \quad (13)$$

where *k* is a constant function of the ship righting arm. Other approximate formulations can be also derived as given by several authors (Meskell, 2011, Rawson et al., 2001) but are not applied here.

5. Test case

The procedures explained in the previous sections are here applied on a decay test performed in a model basin of ahydrodynamic institute. The time series of the recorded data is represented in Fig. 1 where, as usual, the first peak of the record is discarded from the analysis and only the first 6/8 oscillations are considered. In fact the last swings, where the amplitude is significantly small (below 1.5 degrees of amplitude), can affect the quality of the data analysis. For this reason, just the part indicated in the box of Fig. 1 was selected for the analysis.

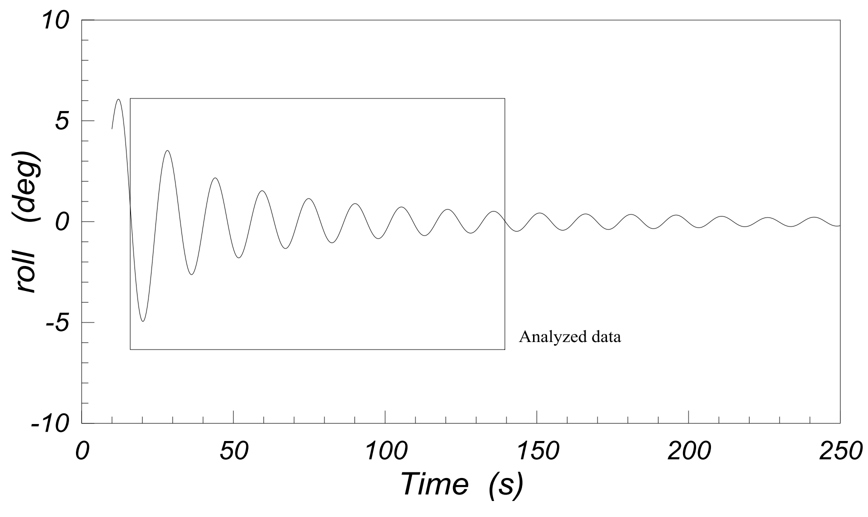

Fig. 1: Time series of the analyzed roll decay test.

As common practice, the analyzed data are presented in graphical form (Fig. 2) in such a way that the linear damping coefficient is represented as function of the swing amplitude or mean amplitude as per the selected method of analysis. In addition, also the quadratic damping coefficient can be determined from the linear regression of the plotted data. In Tab. 1 the damping coefficients according to the different methods are reported.

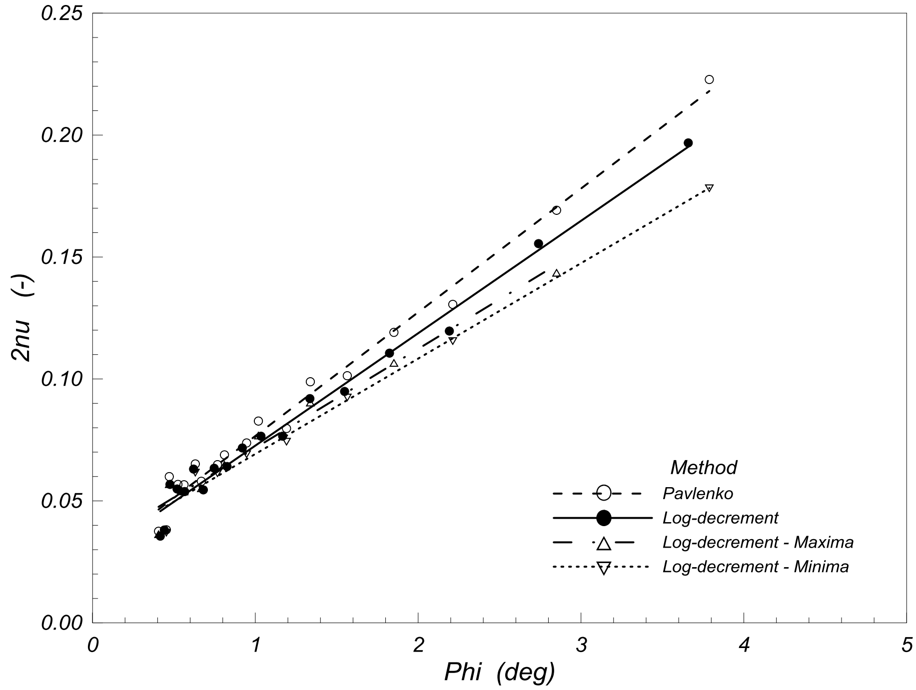

Fig. 2: Graphical comparison of the different analysis methods.

The results in Tab. 1 show that the considered methods are giving comparable results regarding the linear damping coefficient *2ν*. However, due to the different kind of approximations adopted to extract the roll decrements, the quadratic damping part is presenting quite some spread in the obtained data. This can be also seen in Fig. 1, observing the different slopes of the regression curves

Tab. 1: Damping values according to different analysis methods.

Method	2ν	w
Logarithmic decrement	0.027	3.110
Logarithmic decrement (maxima)	0.031	2.742
Logarithmic decrement (minima)	0.030	2.640
Pavlenko	0.026	3.424

6. Conclusions

The classical methods presented in this paper and the derived formulations for the determination of non-linear damping coefficients for the roll motion of a vessel are giving comparable results in the determination of the linear damping coefficient. The quadratic coefficient is suffering a higher spread in the predictions. Once small amplitude motions are analyzed the current methods can be considered satisfactory for the determination of the roll damping coefficients. When non-linearities become of higher order, not only in the damping but also in the righting arm, then it will be necessary to study new methods to overcome to this issue.

References

Blagoveshchensky, S.N. (1964) Theory of ship motions, volume I. Dover Publications Inc., New York.

Blagoveshchensky, S.N. (1964) Theory of ship motions, volume II. Dover Publications Inc., New York.

Meskell, C. (2011) A decrement method for qualifying nonlinear and linear damping in multi-degree of freedom systems. ISRN Mechanical Engineering 2011.

Rawson, K.J. and Tupper, E.C. (2001) Basic ship theory. Butterworth-Heinemann.

Vlasov, V.G. (1930) On the period of free oscillation in rolling. NTK UVMS, Bjul.