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THICK-WALLED COMPOSITE PIPES UNDER BENDING

O. Menshykov[*], **M. Menshykova**[*], **I. A. Guz**[*]

Abstract: *The current study is focused on stress-strain analysis of multi-layered thick-walled fibre reinforced composite pipes (manufactured by filament winding) subjected to bending loading. The problem is solved numerically using the equilibrium equations with the appropriate boundary and interface conditions. Comparative analysis of stresses distribution in the pipes with inner homogeneous layer and inner unidirectional reinforced composite layer is presented.*

Keywords: Fibre-reinforced composite, Thick-walled pipes, Bending load, Stress distribution.

1. Introduction

The formulation of the problem of anisotropic single-layer pipe subjected to bending load was initially given by Lekhnitskii (1981). Later the bending behaviour of thick-walled filament-wound sandwich pipes made of a non-reinforced core layer and alternate-ply skin layers was studied by Xia et al. (2002). Laminated plate theory and Lekhnitskii's stress function approach was used for obtaining the analytical solution for multi-layered filament-wound composite pipes under bending loading. Furthermore the analytical solution for the design of spoolable composite tubes was presented by Starbuck et al. (2000) and the prediction of bending strength and failure modes for filament-wound composite pipes was given by Natsuki et al. (2003). The multi-parametric investigation of stress distribution as a function of the inner layer material properties, its thickness, the number of layers, lay-up and the magnitude of bending load was carried out by Menshykova et al. (2014).

The current study is focused on stress analysis of multi-layered thick-walled fibre reinforced composite pipes subjected to bending loading. The research focuses on the pipes with multi-layered outer part and thin homogeneous inner layer. It provides analytical solution (within the framework developed Natsuki et al. (2003)) and comparative study of stresses distribution in the pipes with inner homogeneous layer and inner composite layer of 0 ° fibre orientation. The investigation of stress distribution as a function of the inner layer material properties was carried out.

2. Stress analysis

Let us consider a multi-layered fibre reinforced filament-wound composite pipe with r_0 inner radius and r_a outer radius subjected to bending load. Each layer of the pipe consists of two laminae with principal material directions symmetrical to the axial direction. Consequently, each layer (two adjacent lay-ups) is assumed to behave as an orthotropic unit. Then for each orthotropic layer we have the constitutive equation for strains in terms of stresses (see Natsuki et al. (2003)):

$$\begin{bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \\ \gamma_{r\theta} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 \\ S_{12} & S_{22} & S_{23} & 0 \\ S_{13} & S_{23} & S_{33} & 0 \\ 0 & 0 & 0 & S_{44} \end{bmatrix} \begin{bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \tau_{r\theta} \end{bmatrix} \tag{1}$$

where S_{ij} are compliance constants and r, θ and z denoted as radial, hoop and axial coordinates. The compliance constants can be obtained from:

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[*] School of Engineering, University of Aberdeen, Aberdeen AB24 3UE, UK

$$(S_{ij}) = (Q_{ij})^{-1}(a_{ij})(P_{ij}), \qquad (2)$$

where matrix (a_{ij}) can be obtained from engineering constants; matrices (P_{ij}) and (Q_{ij}) are the coordinate transformation matrices between the on-axis and the cylindrical axis.

The components of the stress field (Natsuki et al., 2003)) could be presented in the form:

$$\sigma_r = (Br^{-1+\beta} - Cr^{-1-\beta} + A\eta r)\sin\theta, \ \sigma_\theta = [B(1+\beta)r^{-1+\beta} - C(1-\beta)r^{-1-\beta} + 3A\eta r]\sin\theta, \ (3)$$

$$\tau_{r\theta} = -(Br^{-1+\beta} - Cr^{-1-\beta} + A\eta r)\cos\theta, \ \sigma_z = (s_1 Br^{-1+\beta} + s_2 Cr^{-1-\beta} + s_3 Ar)\sin\theta, \quad (4)$$

where A, B, C are the unknown constants and

$$s_{1,2} = \mp\frac{S_{13} + S_{23}(1\pm\beta)}{S_{33}}, \quad s_3 = 1 - \frac{(S_{13} + 3S_{23})\eta}{S_{33}},$$

$$\beta = \sqrt{1 + \frac{R_{11} + 2R_{12} + R_{44}}{R_{22}}}, \quad \eta = \frac{S_{23} - S_{13}}{R_{11} + 2R_{12} + R_{44} - 3R_{22}}, \ R_{ij} = S_{ij} - \frac{S_{i3}S_{j3}}{S_{33}} \quad (i,\, j = 1, 2, 4).$$

Radial and hoop displacements have the following form:

$$u_r = (p_1 Br^{\beta} + p_2 Cr^{-\beta} + p_3 Ar^2)\sin\theta,$$

$$u_\theta = (q_1 Br^{\beta} + q_2 Cr^{-\beta} + q_3 Ar^2)\cos\theta, \qquad (5)$$

and

$$p_{1,2} = \frac{R_{11} + R_{12}(1\pm\beta)}{\beta}, \quad p_3 = \frac{(R_{11} + 3R_{12})\eta + S_{13}}{2},$$

$$q_{1,2} = \frac{R_{11} + R_{12} \mp R_{22}\beta(1\pm\beta)}{\beta}, \quad q_3 = \frac{(R_{11} + R_{12} - 6R_{22})\eta + S_{13} - 2S_{23}}{2}.$$

Material properties in laminated multi-layered tube vary from layer to layer, however it is required that the stress and displacement continuity conditions be satisfied at the layer interfaces (Herakovich, 1998). As the tube is subjected to no inner or outer pressure, the boundary conditions have the following form:

$$\sigma_r^{(1)}(r_0) = 0, \quad \sigma_r^{(N)}(r_a) = 0. \qquad (6)$$

For perfectly bonded layers all displacements must be continuous from layer to layer. For the displacements and stresses on the layer interfaces the continuity conditions are (Xia et al., 2002):

$$u_r^{(k)}(r_k) = u_r^{(k+1)}(r_k), \quad u_\theta^{(k)}(r_k) = u_\theta^{(k+1)}(r_k),$$

$$\sigma_r^{(k)}(r_k) = \sigma_r^{(k+1)}(r_k) \qquad k = \overline{1, N-1}, \qquad (7)$$

where N is the number of layers.

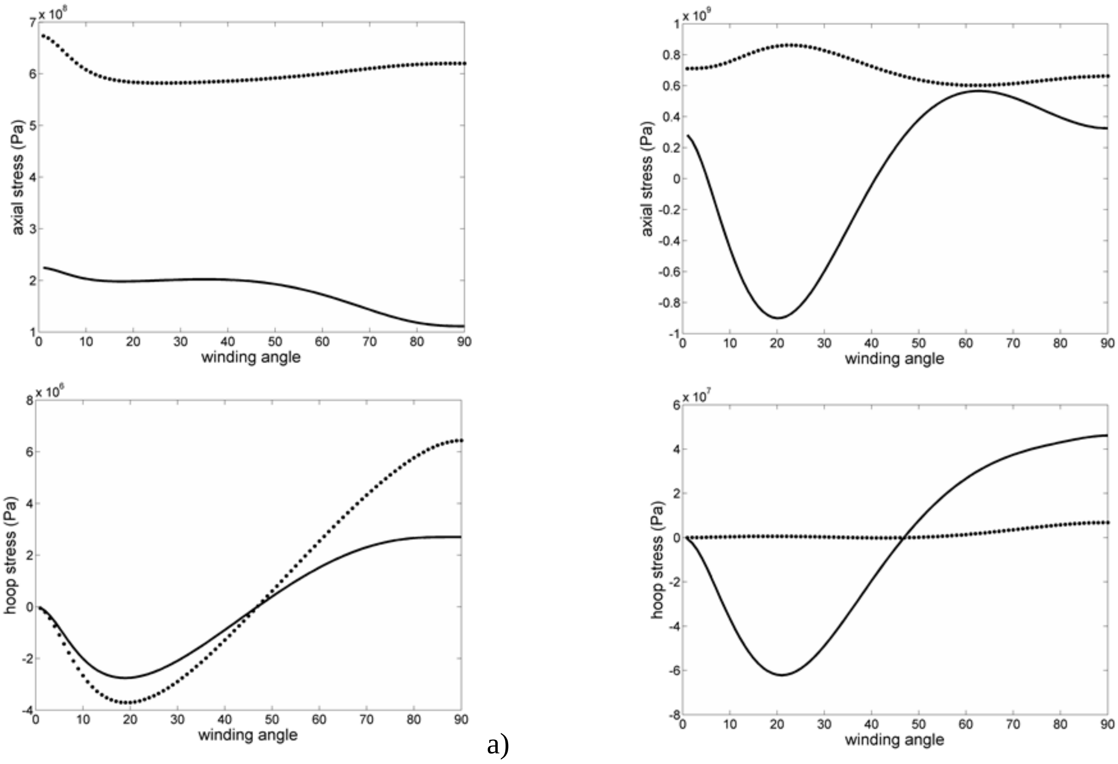
As a result one can derive the system of equations (including the equilibrium equations for bending moment, boundary conditions and continuity conditions for the interfaces, e.g. see Menshykova et al. (2014)) solving which the stress in the tube can be obtained.

3. Numerical results and discussion

To get the numerical results the software for bending stiffness and bending stress calculation was developed. As a numerical example we will consider the filament wound pipes of different designs made of carbon/epoxy composite (T300/LY5052) under bending load. The inner diameter of the pipes is 0.5cm and the outer diameter is 1.5 cm. The investigation of the pipes stiffness change and the comparative study of the stress distribution in the pipes with inner layer of homogeneous material (steel) and unidirectional inner layer of 0^0 fibre orientation was carried out in Menshykova et al. (2014). The properties of steel and composite are given in Tab. 1.

Tab. 1: Properties of materials.

	Carbon/Epoxy unidirectional fibre composite (T300/LY5052) (Bakaiyan et al., 2009)	Steel
E_1 (GPa)	135	205
E_2 (GPa)	8	205
G_{12} (GPa)	3.8	77
ν_{12}	0.27	0.33
ν_{23}	0.49	0.33



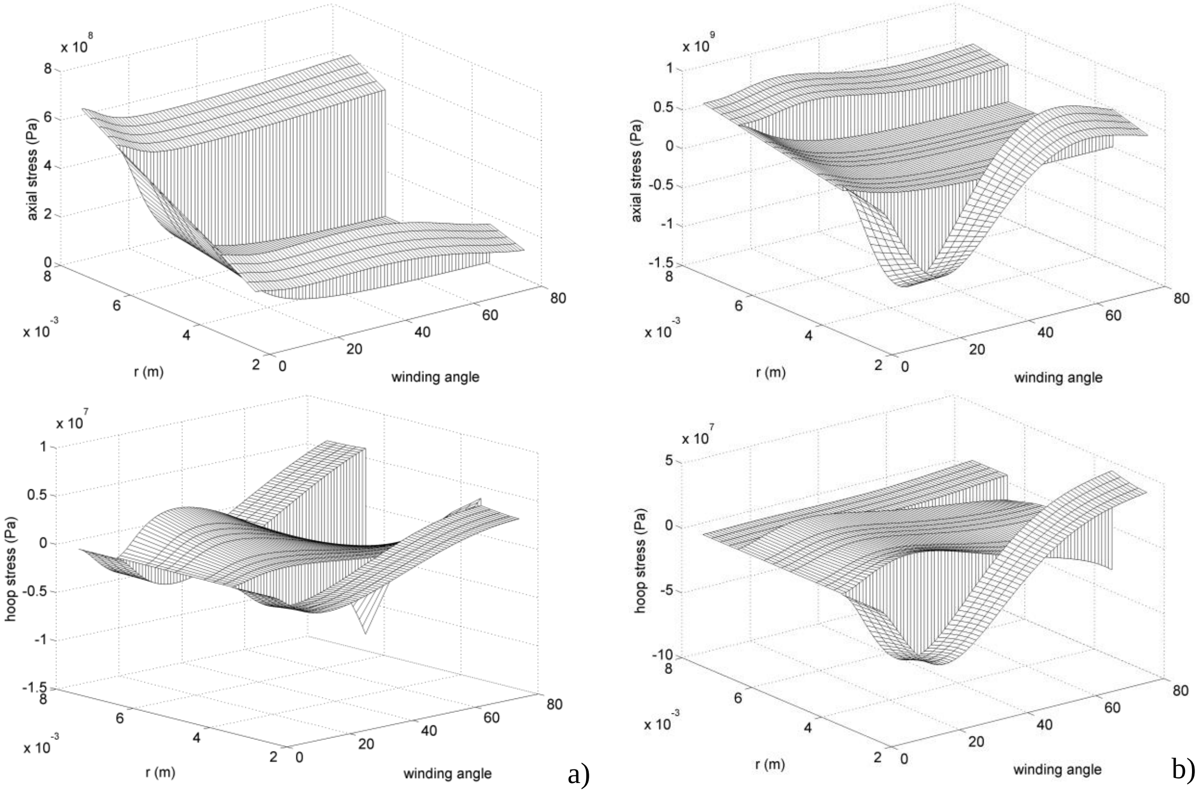
a) b)

Fig. 1: Effect of the winding angle on the axial and hoop stresses for the pipe with a) [0, a, -a, 0]; b) [steel, a, -a, 0] lay-up and layer thicknesses [1 mm, 1.5 mm, 1.5 mm, 1 mm] (solid line – inner radius; dotted line – outer radius) (Menshykova et al., 2014).

The effect of winding angle on axial and hoop stresses on inner and outer pipe surfaces is presented in Figs. 1a and 1b. Axial stresses on inner surface are lower than on outer for pipes with inner steel and inner 0 ° fibre layer. The hoop stress in pipe with inner steel layer is higher on inner surface than on outer for winding angles from 40 ° till 90 ° and lower on inner surface than on outer for winding angles from 0 ° till 40 °. But for the pipe with inner 0 ° fibre layer the situation is vice versa. The hoop stress in pipe with inner 0 ° fibre layer is lower on inner surface than on outer for winding angles from 40 ° till 90 ° and higher on inner surface than on outer for winding angles from 0 ° till 40 °.

Figs. 2a and 2b (Menshykova et al., 2014) provide trough the thickness distribution of axial and hoop stresses for a range or winding angles for the pipes with inner steel and 0 ° fibre layers.

Multi-parametric analysis of the results obtained for various properties of the pipe (thickness, lay-out, reinforcement orientation, etc.) was also carried out.



a) b)

Fig. 2: Effect of the winding angle on the axial, hoop and radial stress distribution through the wall thickness for the pipe with a) [0, a, -a, 0]; b) [steel, a, -a, 0] lay-up, layer thicknesses [1 mm, 1.5 mm, 1.5 mm, 1 mm].

4. Conclusions

Due to the advantages over conventional materials in strength, stiffness, corrosion resistance and the ability to be tailored for the purpose the composite coiled tubing has a potential to replace the conventional steel coiled tubing. Coiled tube is thick walled tube which must withstand various loading conditions one of which is obviously bending loading. Due to the manufacturing issues the inner pipe is used for multi-layered pull winding fibre reinforced tube production. The comparative study of stress distribution in multi-layered fibre reinforced pipes with inner steel layer and inner 0 ° fibre layer is presented. The analysis shows how the stresses depend on the material of inner layer. Multi-parametric analysis of the results obtained for various properties of the pipe (thickness, lay-out, reinforcement orientation, etc.) was also carried out.

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