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MODELLING AND MEASUREMENT OF NON-UNIFORM ELASTOSTATIC TORSION OF HOLLOW CROSS-SECTION BEAMS

J. Murín[*], V. Goga[**], V. Kutiš[***], J. Hrabovský[****], T. Sedlár[*****], J. Paulech[******]

Abstract: *This paper presents results from measurements of hollow thin-walled cross-section beam under the non-uniform torsion conditions where the warping effect is evaluated and compared with the numerical analysis results using new finite element (Murín et al., 2014) and with results from finite element analysis by program ANSYS.*

Keywords: Non-uniform torsion, Modeling and simulation, Experimental verification.

1. Introduction

In stress and deformation analyses of thin-walled structures subjected to torsion, the effect of warping must be considered. Extreme values of axial stress, caused by the bimoment, mainly occurs at the points of action of concentrated torsion moments (except for free ends) and at sections with warping restraints.

Special theories of torsion with warping, usually referred to as non-uniform torsion or warping torsion were used to solve such problems analytically (e.g. Vlasov, 1961). The analogy between the 2nd order beam theory with axial tension and torsion including warping has also often been exploited (e.g. Roik et al., 1966 and Rubin, 2005). However, it is worth of note that in the literature and in engineering practice, as well as in the guidelines provided by Eurocode 3 (EN 1993 – Eurocode 3, 2004), the significance of the effect of warping is assumed to be restricted to open cross-sections.

Warping based stresses and deformations in hollow cross-section (HCS) are assumed to be insignificant and have, therefore, generally been neglected. According to the aforementioned theory of torsion of open cross-sections including warping and according to the mentioned analogy, special finite beam elements were designed and implemented into finite element codes (e.g. ANSYS and RSTAB, 2006). Important progress in the solution of torsion with warping is documented in (Sapountzakis et al., 2007), where a combination of the boundary element method (BEM) and the finite element method (FEM) was used, allowing warping analysis of composite beams with a longitudinally varying cross-section. There are several other papers, e.g. (Mokos et al., 2004, Kim et al., 2005, Genoese et al., 2013 and Minghini et al., 2007), which deal with the problem of non-uniform torsion. However, recent theoretical results have shown that the effect of warping must also be considered for the case of HCS beams (Rubin, 2006). Based on recent research concerning the aforementioned analogy, reported in (Rubin, 2006, Rubin, 2007, Rubin et al., 2007 and Aminbaghai et al., 2012), the local stiffness relation of a new two-node finite element for torsion including warping of cross-sections of straight beams was derived (Murín et al., 2014). This new finite element can be used for non-uniform torsion analysis of straight beams with open

[*] Prof. Ing. Justín Murín, DrSc.: Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology, STU in Bratislava, Ilkovičova 3; 812 19, Bratislava; Slovakia, justin.murin@stuba.sk

[**] Assoc. Prof. Ing. Vladimír Goga, PhD.: Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology, STU in Bratislava, Ilkovičova 3; 812 19, Bratislava; Slovakia, vladimir.goga@stuba.sk

[***] Assoc. Prof. Ing. Vladimír Kutiš, PhD.: Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology, STU in Bratislava, Ilkovičova 3; 812 19, Bratislava; Slovakia, vladimir.kutis@stuba.sk

[****] Ing. Juraj Hrabovský, PhD.: Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology, STU in Bratislava, Ilkovičova 3; 812 19, Bratislava; Slovakia, juraj.hrabovsky@stuba.sk

[*****] Ing. Tibor Sedlár:Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology, STU in Bratislava, Ilkovičova 3; 812 19, Bratislava; Slovakia, tibor.sedlar@stuba.sk

[******] Ing. Juraj Paulech, PhD.: Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology, STU in Bratislava, Ilkovičova 3; 812 19, Bratislava; Slovakia, juraj.paulech@stuba.sk

as well as hollow cross-sections. This paper presents results from measurements of hollow thin-walled cross-section beam under the non-uniform torsion conditions where the warping effect is evaluated and compared with the numerical analysis results using new finite element (Murín et al., 2014) and with results from finite element analysis by program ANSYS.

2. Experimental measurements

The original experimental device (Sedlár et al., 2013) for the measurement of warping effects in thin-walled beams were designed, see Fig. 1. Device allows an arbitrary torque position plus a variety of boundary conditions at various beam positions (e.g. at beam's ends). The twist angle, the free-end cross-section warping and the normal stress caused by bimoment can be measured.

Measuring device characteristics: beam sample positioning: horizontal; boundary conditions - supports: free, fixed, and forked; advantages: variable beam length, variable beam supports, torque application at various positions; disadvantage: complicated equipment for ideal torque application.

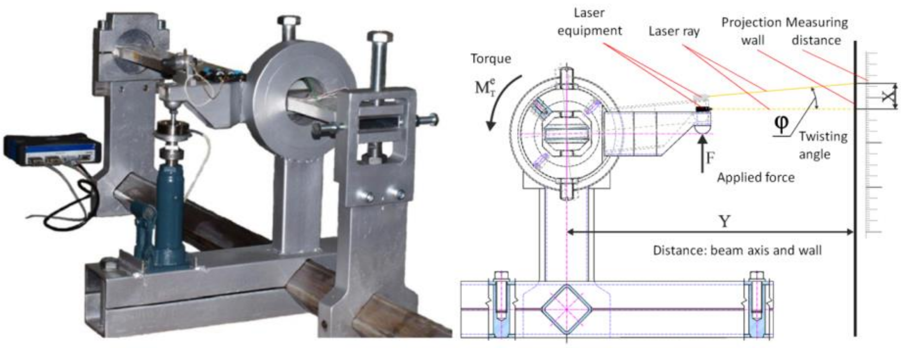

Fig. 1: Measuring device.

The aluminium beam with hollow rectangular cross-section were loaded with a torque of M_T = 100 Nm. Material properties of the beam are: the Young modulus E = 68 GPa, the shear modulus G = 26 GPa, the Poisson's ratio $\nu = 0.33$. The beam length L = 980 mm. Dimensions of the rectangular hollow cross-section are: the height H = 40 mm, the width B = 20 mm, the wall thickness t = 2 mm. Both ends of the beams are clamped and the torque was applied in the middle of the beam span.

Expected normal stress distribution along the beam axis is based on the theory of non-uniform torsion (Vlasov, 1961, Roik et al., 1966 and Rubin, 2005), see Fig. 2 (left). This normal stresses were measured using six strain gauges. Positions of the strain gauges (signed T_1-T_6) are shown in Fig. 2 (right).

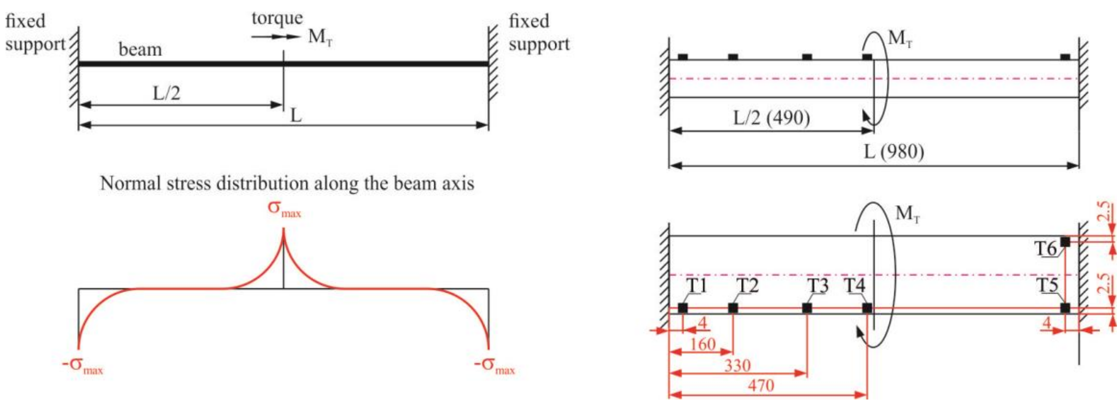

Fig. 2:Theoretically expected bimoment normal stress distribution along the beam edge (left), the strain gauges positioning (right).

3. Finite element calculation

The non-uniform torsion of the measured beam was also solved using the SOLID186 finite element of the program ANSYS and by new warping beam finite element (WB) (Murín et al., 2014). In the case of the SOLID186 model, the beam was modeled in the half length. The left beam’s end is clamped and the symmetrical boundary conditions are applied at the right beam’s end. At the right beam’s end the half torque was applied through the rigid steel flanges with frictionless contact. By this way, the accordance of the measured and the calculated model is relatively well obtained. The finite element model was created using very fine mesh of 50000 SOLID186 elements, see Fig. 3. Only 2 WB finite elements are used in the calculation of whole beam by our approach (Murín et al., 2014). The SOLID186 finite element result for normal stress distribution along the beam edge is shown in Fig. 4. As shown in Fig. 4, a difference occurs between the expected theoretical and the calculated distribution.

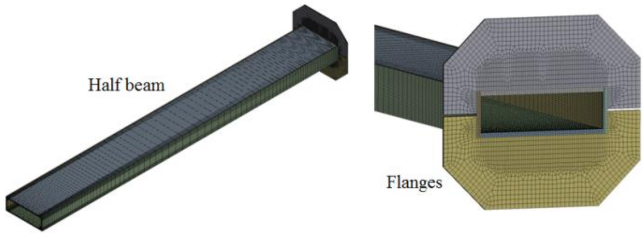

Fig. 3: The finite element model (SOLID186).

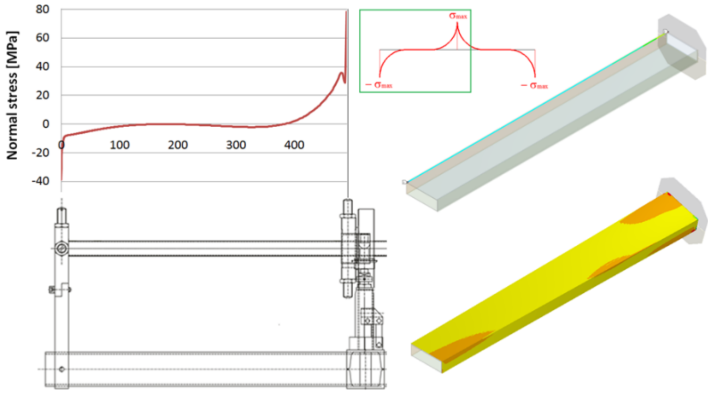

Fig. 4: The calculated (by SOLID186) and the theoretical (red line) normal stress distribution.

Tab. 1 present results for the maximal normal stress on the beam edge at the positions where the strain gauges were placed (Fig. 2). The differences between the measurement and numerical calculations are denoted as $\Delta_{Me\text{-}WB}$ – for WB, and $\Delta_{Me\text{-}An}$ for ANSYS.

Tab. 1: Measured and calculated results.

Torque = 100 Nm					
Position	Normal stress				
	Measurement	WB (Murín et al., 2014)	ANSYS	$\Delta_{Me\text{-}WB}$	$\Delta_{Me\text{-}An}$
	[MPa]	[MPa]	[MPa]	[%]	[%]
T_1	-12.04	-12.49	-11.56	-3.7	4.0
T_2	-1.97	-0.54	-1.75	72.6	11.2
T_3	-1.69	0.21	-1.85	112.4	-9.5
T_4	25.86	5.86	25.47	77.3	1.5
T_5	-12.47	-12.49	-11.56	-0.2	7.3
T_6	12.54	12.49	11.56	0.4	7.8

Outputs of the measurements and numerical analysis show significant effect of the warping on the normal bimoment stress by non-uniform torsion of beam with closed thin walled cross-section. This finding is in contrary to the statement in EN 1993 – Eurocode 3 that influence of the non-uniform torsion can be neglected by twisted beams with closed cross-sections. The measured and SOLID186 results are in good agreement each to other. But the WB results differ significantly at the acting point of the torque from the other ones. This difference is caused by the way, how the torque is applied in the real model and in the 3D-solid finite element model. In the WB finite element model, the torque acts on the longitudinal beam axis but in reality as well in the SOLID186 model the torque is realized by contact between the rigid flanges and the beam cross-section surfaces. This contact produces local stress concentration due to contact pressure. If the task is solved with the BEAM188 (warping restraint or unrestrained, ANSYS), the results are not relevant.

4. Conclusions

Numerical analyses of the rectangular hollow cross-section beams loaded by non-uniform torsion are performed. The SOLID186 (ANSYS) and warping beam (WB) (Murín et al., 2014) finite elements are used for evaluation of the normal bimoment stress and the twist angle. Experimental verification of the numerical results is done. From the results comparison, following conclusions can be drawn:

- normal stress caused by non-uniform torsion of the rectangular hollow cross-section beams are not neglectable as stated in EN 1993 – Eurocode 3;
- in opposite to standard warping beam finite elements, the results obtained by our new warping beam finite element (WB) (Murín et al., 2014) agree well with the measured results except the case when the boundary conditions and load action do not fulfil the appropriated beam theory;
- realisation of the boundary conditions and the load action affect the stress state significantly what leads to differences in calculated and measured results.

Acknowledgement

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