23rd International Conference
ENGINEERING MECHANICS 2017
Svratka, Czech Republic, 15 – 18 May 2017

NON-HOLONOMIC PLANAR AND SPATIAL MODEL OF A BALL-TYPE TUNED MASS DAMPING DEVICE

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Abstract: *The area of tuned mass dampers is a wide field of inspiration for theoretical studies in non-linear dynamics and dynamic stability. The studies attempt to estimate behaviour of diverse damping devices and their reliability. The current paper deals with the response of a heavy ball rolling inside a spherical cavity under horizontal kinematic excitation. The non-linear system consists of six degrees of freedom with three non-holonomic constraints. The contact between the ball and the cavity surface is supposed to be perfect without any sliding. The mathematical model using the Appell-Gibbs function of acceleration energy is developed and discussed. Comparison with previous planar (SDOF) model which is based on the Lagrangian procedure is given. The system has an auto-parametric character and therefore semi-trivial solutions and their dynamic stability can be analysed. The most important post-critical regimes are outlined and qualitatively evaluated in resonance domain. Numerical experiments were performed when excitation frequency is slowly swept up and down to identify different modes of response. Some applications in civil engineering as a tuned mass damper, which can be used on slender structures, are mentioned. The proposed device is compared with a conventional pendulum damper. Strengths and weaknesses of both absorbers types are discussed.*

Keywords: Tuned mass damper, Moving ball, Appell-Gibbs formulation.

1. Introduction

Various types of passive vibration absorbers are regularly used in civil engineering for suppression of wind induced vibration. Transmission towers, TV masts and other slender structures exposed to wind excitation are usually equipped by such devices. Usual pendulum-style passive absorbers, see, e.g., (Haxton, 1974), utilize the auto-parametric resonance for their damping effect. Although they are very effective and reliable their application can be limited by several disadvantages. Dimensions of the pendulum and namely its suspension length cannot be neglected or minimized and it could easily happen that the structure cannot accommodate this device. This is particularly true for existing structures, where an absorber should be installed as a supplementary equipment. Also horizontal constructions, like foot bridges, usually cannot include a pendulum-style absorber. Moreover, the complete installation has to remain accessible to allow a regular maintenance.

The ball-type absorber represents an alternative solution, which is less spatially-demanding and practically maintenance-free. The basic principle comes out of a rolling movement of a metallic ball of a radius r inside of a metallic rubber coated spherical cavity of a radius $R > r$, Fig. 1a. The system can be closed in an airtight case. Its vertical dimension depends only on the dimension of the rolling ball and thus the assembly can be relatively very small. Such device can be used in cases where a pendulum absorber is inapplicable due to lack of vertical space or difficult maintenance.

First papers dealing with the theoretical and practical aspects of ball absorbers have been published by Pirner and Fischer (1994, 2000). The first analysis of the problem on the basis of the rational dynamics has been published by the authors in (Náprstek and Pirner, 2002) and later extended in (Náprstek et al., 2011). The approach in the referenced papers was based on planar model, constructed using the Hamiltonian functional with non-holonomic constraints and the respective Lagrangian governing system

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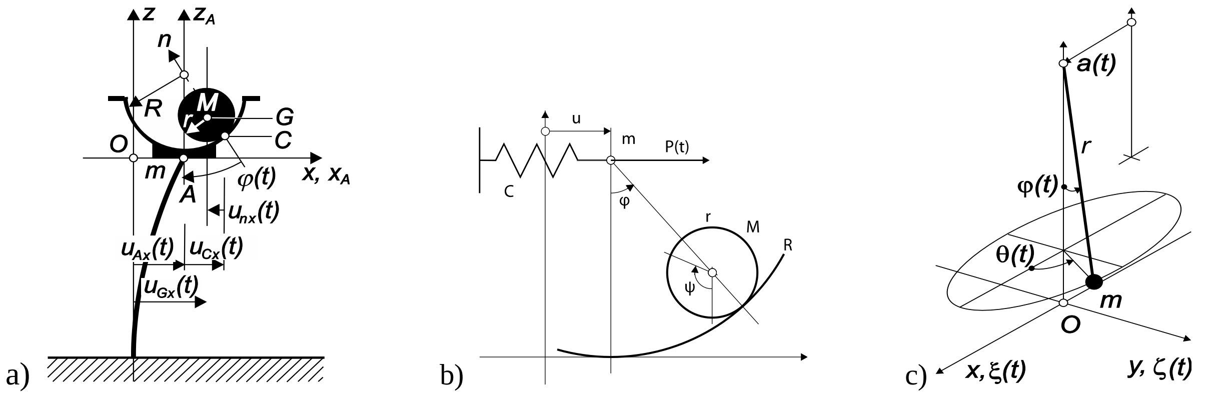

Fig. 1: a) Outline of the ball absorber; b) Scheme of the simplified 2D model; c) The spherical pendulum.

in 2D, cf. Fig. 1b. The theoretical derivation together with its numerical evaluation was compared to practical application up to the state of the realization including some results of long-term measurements.

Dynamics of the real ball absorber is significantly more complicated in comparison to the pendulum one, Fig. 1c. Movement of the ball cannot be described in a linear state although for the first view its behaviour is similar to the pendulum absorber type. A number of problems that are related with movement stability, auto-parametric resonances, etc., originate from the spherical cavity and ball surface imperfections. The ball moving inside the spherical cavity is very sensitive to the stability loss of its movement in forcing direction. However, this type of motion is requested, as it determines efficiency of the damper. Due to probability of the stability loss, which is much higher than of the spherical pendulum, semi-trivial states should be carefully analysed including a large variety of post-critical processes.

The fully spatial model, unlike the 2D approximation, does not allow the usual approximations of the exact formulations. The deflection φ of the pendulum, cf. Fig. 1c, can be assumed relatively small, it is usually lower than $20-30\,^{\circ}$, and the approximations in form of a short Taylor series are acceptable: $\sin\varphi \approx \varphi - 1/6\varphi^3, \cos\varphi \approx 1 - 1/2\varphi^2$. On the other hand, movement of the ball within the cavity should respect the full expressions due to the fact that the ball deflection can reach nearly the "equator" of the cavity. This fact prevents to get through the matter by an analytical way, but suitable combination of both numerical/analytical procedures is still possible and, moreover, the model presented in this contribution does not include any limits of the response amplitudes.

Authors tried to formulate this problem in the past by a classical way constructing the Hamiltonian functional with non-holonomic constraints. However, the resulting Lagrangian governing system provides the differential system which is too complicated and its physical interpretation can be multivalent. For easier analysis is the problem formulated using Appell-Gibbs function. Its main advantage consists in easier problem definition and more transparent introduction of non-holonomic constraints.

2. Mathematical models

2.1. Simple planar model

The dynamic character of the complete structure is modelled by a linear SDOF system represented by a total mass m (which includes the structure, cavity and the ball) and stiffness C. The ball with mass M is moving freely in a vertical plane in a cavity directly attached to the structure, i.e., 2DOF system should be investigated as it is outlined in Fig. 1b. For full derivation see (Náprstek et al., 2011).

$$\ddot{\varphi} + \kappa b_{\varphi}\dot{\varphi} + \kappa\omega_M^2 \sin\varphi + \kappa\ddot{\zeta} \cdot \cos\varphi = 0 \qquad \omega_M^2 = \frac{g}{\rho}\,, \quad \kappa = \frac{Mr^2}{J+Mr^2}, \qquad (1a)$$

$$\mu\ddot{\varphi}\cos\varphi - \mu\dot{\varphi}^2\sin\varphi + (1+\mu)\ddot{\zeta} + b_u\dot{\zeta} + \omega_M^2\zeta = p(t) \qquad \mu = \frac{M}{m}\,,\ \omega_m^2 = \frac{M}{m}. \qquad (1b)$$

Here g it the gravitational acceleration and $J = 2/5 \cdot Mr^2$ is the central inertia moment of the ball.

The theoretical efficiency of the absorber can be assessed using its frequency characteristics for excitation by harmonic force $p(t)$. However, to relate the planar model to the fully spatial one, only the first equation (1a) regarding the ball rolling in the cavity will be used here. The action of the elastic structure is replaced by the kinematic excitation $\ddot{\zeta} = \zeta_0 \cdot \sin\omega t$.

2.2. Appell-Gibbs approach to the full spatial model

In the spatial model of the ball absorber (Fig. 1a) is the Appell-Gibbs approach used to formulate the governing non-linear differential system. The basis is the Appell function (often referred to as an energy acceleration function), which is defined as a function of six components characterizing motion of the stiff body in 3D:

$$S \;=\; \frac{1}{2}M(\ddot{u}_{Gx}^2+\ddot{u}_{Gy}^2+\ddot{u}_{Gz}^2)+\frac{1}{2}J(\dot{\omega}_x^2+\dot{\omega}_y^2+\dot{\omega}_z^2), \tag{2}$$

where M is the mass of the ball, $J = 2/5 \cdot Mr^2$ central inertia moment of the ball with respect to its centre, $\omega_{\_}$ angular velocities of the ball with respect to its centre, $u_{\mathrm{G}\_}$ displacements of the ball centre with respect to absolute origin O, $u_{\mathrm{C}\_}$ displacements of the contact point with respect to origin O, and $u_{\mathrm{A}\_}$ prescribed movement of the cavity with respect to origin O.

Following the detailed derivation of the equations of motion the governing system reads (Náprstek and Fischer, 2016)

$$\dot{u}_{Cx} = \omega_y(u_{Cz}-R)-\omega_z u_{Cy} \tag{3a}$$
$$\dot{u}_{Cy} = \omega_z u_{Cx}-\omega_x(u_{Cz}-R) \tag{3b}$$
$$\dot{u}_{Cz} = \omega_x u_{Cy}-\omega_y u_{Cx} \tag{3c}$$
$$J_s\rho\dot{\omega}_x = -\left(\ddot{u}_{Ay}+\rho(\omega_z\dot{u}_{Cx}-\omega_x\dot{u}_{Cz})\right)(u_{Cz}-R)-u_{Cy}\left(g+\rho\left(\omega_x\dot{u}_{Cy}-\omega_y\dot{u}_{Cx}\right)\right)-\rho D_{Gx}/M \tag{4a}$$
$$J_s\rho\dot{\omega}_y = -\left(\ddot{u}_{Ax}+\rho\left(\omega_y\dot{u}_{Cz}-\omega_z\dot{u}_{Cy}\right)\right)(u_{Cz}-R)+u_{Cx}\left(g+\rho\left(\omega_x\dot{u}_{Cy}-\omega_y\dot{u}_{Cx}\right)\right)-\rho D_y/M \tag{4b}$$
$$J_s\rho\dot{\omega}_z = -\left(\ddot{u}_{Ax}+\rho\left(\omega_y\dot{u}_{Cz}-\omega_z\dot{u}_{Cy}\right)\right)u_{Cy}-\left(\ddot{u}_{Ay}+\rho(\omega_z\dot{u}_{Cx}-\omega_x\dot{u}_{Cz})\right)u_{Cx}-\rho D_y/M \tag{4c}$$

where: $J_s = \;(J+M\rho^2\,R^2)/(M\rho^2),\;\rho = 1-r/R$ and terms $D_{G\_}$ cover influence of damping.

Damping in the contact point has to be treated separately for rolling and spinning component. Supposing that no slipping arises in the contact, the dissipation process can be approximated as proportional to the relevant components of the angular velocity. Thus, the damping terms in Eqs. (4) include coordinate transformation from local coordinates of the ball distinguishing rolling and spinning movement.

2.4. Comparison of the models

Although the models aim to describe a single real system, their difference is more than dimensional settings. The simplified model was set up to describe interaction of the ball absorber and the structure, its auto-parametric character originates from coupling between the ball, cavity and elastic structure. The spatial model in the current state describes only the movement of the ball in the cavity and assumes only the kinematic excitation of the cavity. The auto-parametric character follows from interaction of the individual x and y components. Hence only the equation (1a) from the planar model can be directly related to the 3D model.

In order to show correspondence of the full model (3, 4) and Eq. (1a) the following conditions will be assumed, which assure the planar movement of the ball in the cavity:

$$u_{Cy} = 0, \omega_x = 0, \omega_z = 0 \tag{5}$$

Introducing conditions (5) into Eqs (3,4) (without damping), three equations are fulfilled trivially and only Eqs (3a,3c,4b) remain active. Elimination of $\dot{\omega}_y$ from Eq. (4b) using the derived Eq. (3a) and substitution of the geometric relation $u_{Cz} = R-\sqrt{(R^2-u_{Cx}^2}\,)$ gives the single equation which involves only u_{Cx}:

$$\ddot{u}_{Cx} = \frac{M\rho}{J+M\rho^2R^2}\left((u_{Cx}^2-R^2)\ddot{u}_{Ax}-gu_{Cx}\sqrt{R^2-u_{Cx}^2}\right)-\frac{u_{Cx}\dot{u}_{Cx}^2}{R^2-u_{Cx}^2} \tag{6}$$

Transformation of the translational motion to rotation $u_{Cx} = R\sin\varphi$ restores the equation (1a).

Performance of both models is compared in Fig. 2. The system parameters used: $M = 1, R = 1, r = 1/2$, the excitation amplitude $\zeta_0 = 0.1$ and damping: $b_\varphi = \alpha = 0.1, \beta = 0.01$. Symbols α, β are the damping coefficients in the spatial model for rolling and spinning, respectively. The left plot shows the positions of the turning points in lateral direction depending on the excitation frequency ω. Because the ball in the maximal motion crosses the equator of the cavity, the actual extremal value of the coordinate can become

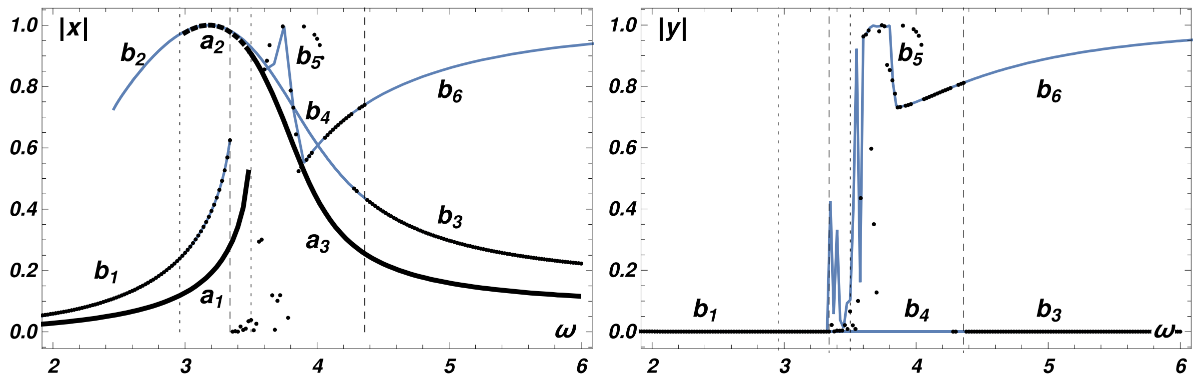

Fig. 2: Resonance behaviour of planar and spatial models. Maximal responses in lateral (left) and transversal (right) direction. Planar model – thick lines, spatial model – thin lines and dots.

smaller for increasing amplitudes (cf. concave part of the curve in the left plot). The resonance curve corresponding to the planar model is shown in the left plot as the thick solid line $a_1 - a_3$ with thick dotted overhanging part a_2. The thin dotted vertical lines delimit area where the resonance curve shows two stable solutions, lines a_1, a_2. The curves $b_1 - b_6$ in the both plots correspond to the spatial model. The parts $b_1 - b_4$ show planar (semi-trivial) solution which can be directly related to the result of the planar model. The branches b_5 and b_6 correspond to spatial periodic (b_6) or chaotic (b_5) movement of the ball, as can be seen comparing both plots. It is worth to note that the periodic branch b_6 continues over right border of the plot. It represents stable cycling which approaches equator of the cavity for increasing excitation frequency. From the left part of Fig. 2 can be seen that the planar model underestimates response and width of the resonance area. Moreover, it cannot encompass the upper spatial branch of the response (b_6), which can have devastating effect on the structure.

3. Conclusions

Two approaches to modelling of behaviour of the ball-style tuned mass damper were presented and compared. Whereas the non-linear planar approach models the ball in the cavity using a single DOF, the spatial one comprises six degrees of freedom with three non-holonomic constraints. The equations of the motion in the spatial model of the ball are derived using the Appell-Gibbs function of acceleration energy. The Appell-Gibbs formulation of a non-holonomic system dynamics approved excellent efficiency in comparison with a conventional way being based on Lagrangian differential system and non-holonomic constraints adjoined via indefinite Lagrange multipliers. The resulting system has an auto-parametric character, it permits to formulate the semi-trivial (planar) solution. The models were numerically analysed with respect to harmonic horizontal excitation. The interval of frequencies leading to instability of the semi-trivial solution was shown and studied and its dangerous character was pointed out.

Acknowledgement

The kind support of the Czech Science Foundation project No. 15-01035S and the institutional support RVO 68378297 are gratefully acknowledged.

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