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DETERMINATION OF THE NUMBER OF CYCLES FOR ASYNCHRONOUS PERIODIC MULTIAXIAL LOADINGS USING ENERGY BASED CUMULATIVE DAMAGE THEORY

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Abstract: *The problem of definition of loading's cycle and fatigue life in case of asynchronous loadings is not fully solved in the literature. A few different approaches can be found, but none of them is widely accepted. In this work, an energy based definition of loading's cycle for asynchronous loadings is proposed. The method uses plastic strain energy density of complex loading's components as a weightening factors to calculate weighted arithmetic mean of loading's components lives.*

Keywords: Multiaxial fatigue, Asynchronous loadings, Cumulative damage, Fatigue life prediction.

1. Introduction

In multiaxial fatigue testing of materials, the experimental fatigue lives are usually used to analyse achieved results using two methods:

– the experimental fatigue life is compared for different types of loadings, which result in the same value of selected fatigue damage parameter; usually, damage parameter–fatigue life curves are used (Mei and Dong, 2017, 2016, Pejkowski et al., 2014a, 2014b; Skibicki et al., 2014) (Fig. 1a),

– a damage parameter is calculated for different types of loading, and on its basis the fatigue life is predicted by inversion of damage parameter–fatigue life equation for uniaxial loading (Carpinteri et al., 2016); next, calculated–experimental fatigue life diagrams ($N_{f,cal}$ – $N_{f,exp}$) are usually generated (Böhm and Niesłony, 2015; Karolczuk, 2016; Pejkowski, 2016; Pejkowski et al., 2012) (Fig. 1b).

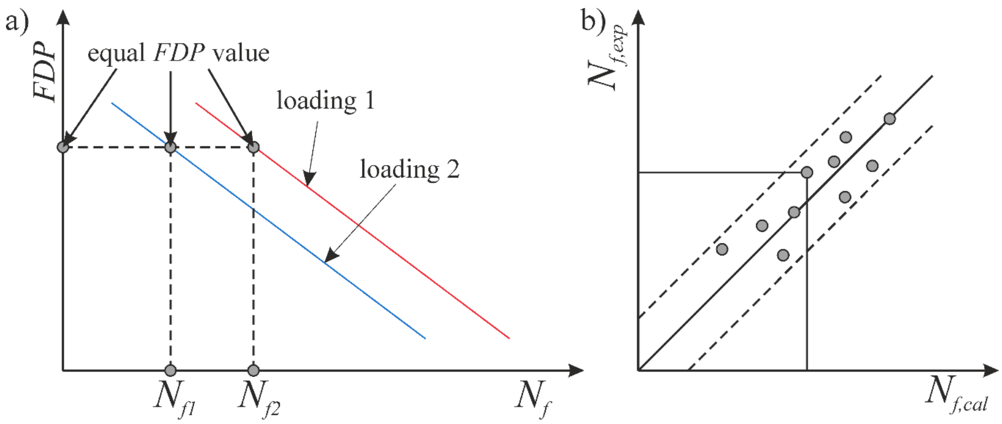

Fig. 1: a) Comparison of experimental fatigue lives for two loadings of the same FDP value; b) Comparison of experimental and calculated fatigue lives.

Comparison of calculated fatigue lives with experimental ones is unequivocal in case of any uniaxial, complex proportional or out-of-phase non-proportional loadings, since their components have the same frequencies, and thus their numbers of cycles to failure are equal.

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Another case of non-proportional loadings, that is also being often analysed, are asynchronous loadings for which a difference in frequencies of components is present. An example of asynchronous loading with strain frequency ratio $f_\gamma/f_\varepsilon = 0.5$ is shown in Fig. 2.

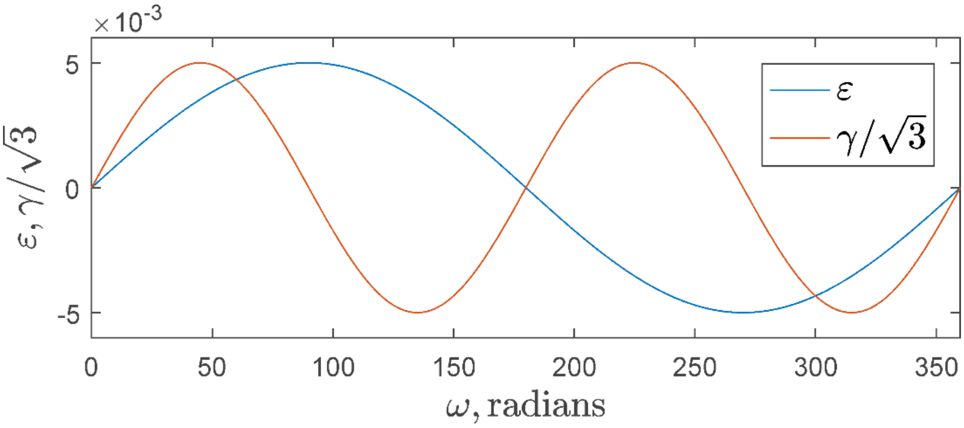

Fig. 2: An example of asynchronous loading.

A term "experimental fatigue life" becomes ambiguous, since each loading's component reaches different number of cycles at failure. The number of cycles cannot also be counted using simple damage parameter such as Huber-von Mises equivalent stress or strain. And here comes the question: what is the fatigue life in case of this case of loadings? To compare fatigue lives in case of asynchronous loadings with fatigue life for axial loading, which is usually used as basic, it is fundamental problem to answer this question.

In (Anes et al., 2014), Anes et al. showed that cycle counting of SWT or Fatemi-Socie damage parameter on critical plane using popular Bannantine and Socie method or Wang and Brown damage parameter and cycle counting method, which are dedicated to multiaxial loadings, would result in errors in case of asynchronous loadings. An observation of the problem with loading's cycle definition in case of loadings being discussed, has been also done by Dong et al. in (Dong et al., 2010; Mei and Dong, 2017, 2016). They aptly pointed that many authors ignore this problem and proposed path-dependent cycle counting method, which seems to be more reasonable.

2. Energy based cycle definition

Jahed, Noban et al. (Jahed et al., 2007; Noban et al., 2011), proposed a simple linear rule to calculate the fatigue life of complex loadings. Fatigue lives are first calculated on the basis of total energy value by introduction into equations for axial and torsional loadings separately. Then, following formula is used to estimate final fatigue life:

$$N_f = \frac{\Delta W_A}{\Delta W_T} N_A + \frac{\Delta W_S}{\Delta W_T} N_S, \tag{1}$$

where N_f is calculated fatigue life, N_A and N_S are the numbers of cycles calculated using equations for axial and torsional loadings, respectively and ΔW_T, ΔW_A and ΔW_S are total, axial and shear elastic–plastic strain energies.

Based on this approach, the generalized, "equivalent fatigue life" term can be introduced. It is defined as a weighted arithmetic mean:

$$N_f = \frac{\sum_{i=1}^{n} W_i N_i}{\sum_{i=1}^{n} W_i}, \tag{2}$$

where W_i is the plastic strain energy density and N_i is a number of cycles of complex loadings component.

In (Pejkowski et al., 2016) part of series of multiaxial fatigue test conducted on CuZn37 brass was presented. Among the others, two cases of asynchronous loadings were utilized. For the first one $f_\gamma/f_\varepsilon = 0.5$ and for the second one $f_\gamma/f_\varepsilon = 2$. Numbers of cycles for one common period of loading's components (for complete loading path in other words) are equal to $N_A = 2$ and $N_S = 1$ for $f_\gamma/f_\varepsilon = 0.5$ and $N_A = 1$ and $N_S = 2$ for $f_\gamma/f_\varepsilon = 2$. To estimate the equivalent fatigue life for these loadings, two axial and one shear hysteresis loops energy have to be introduced to eq. (2) in the first case and conversely. For both loading cases this approach results in number of cycles for one common period equal approximately to 1.6. It is worth to notice that this number is close to values obtained by Anes et al. (Anes et al., 2014) by their own proposal, used for high cycle, stress controlled tests. For f_γ/f_ε = 0.5 these author determined 1.8 cycles and for f_γ/f_ε = 2 respectively 1.4 cycles.

The results of presented approach for fatigue life estimation, for the two cases of loadings being discussed are presented in Fig. 3. Results for tension-compression (TC) are introduced for comparison. Power function was used to fit the results for TC.

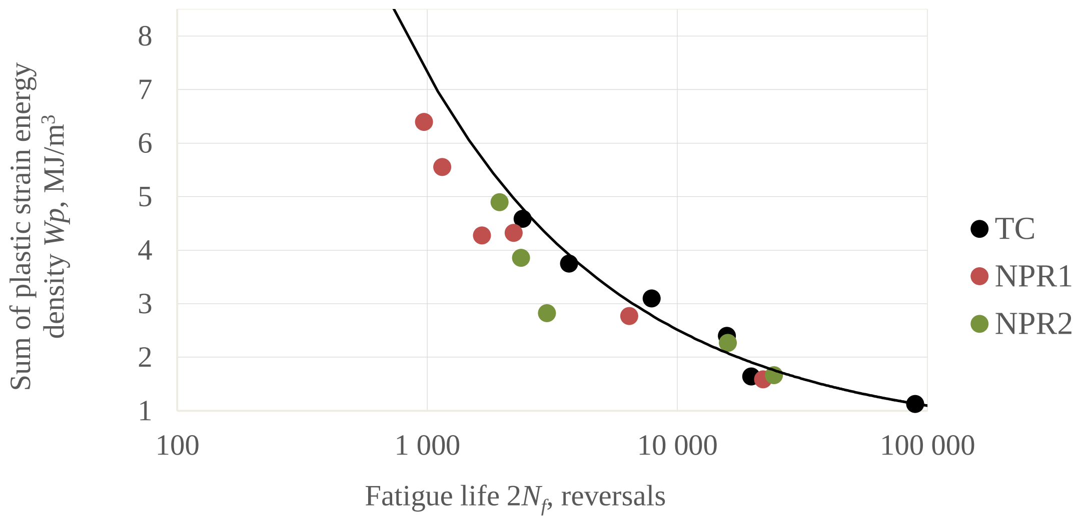

Fig. 3: Correlation of total plastic strain energy density with fatigue life; NPR1: loading with f_γ/f_ε = 0.5 and NPR2: loading with f_γ/f_ε = 2.

A very good correlation with fatigue life, determined using the above described method, can also be observed for cumulated total plastic strain energy density to fracture (Fig. 4). The loading's sequence effect is not present in case of asynchronous loadings, thus the "equivalent fatigue life" coupled with energy based cumulative damage theory can be used for fatigue life prediction.

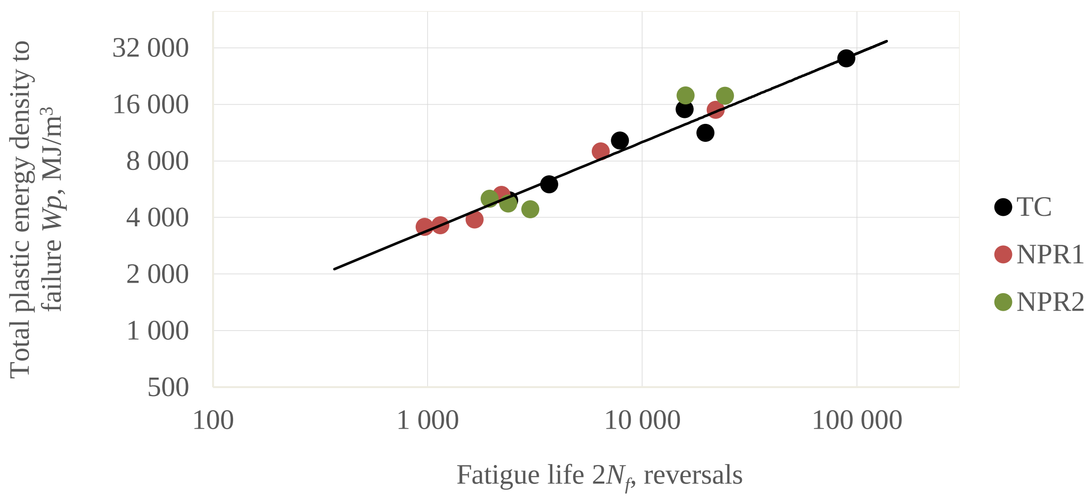

Fig. 4: Correlation of cumulated total plastic strain energy density to fracture with fatigue life; NPR1: loading with f_γ/f_ε = 0.5 and NPR2: loading with f_γ/f_ε = 2.

3. Conclusions

The "equivalent fatigue life" method based on Jahed, Noban et al. approach was used for asynchronous loadings to define and determine the experimental fatigue life for one loading path. It seems to be a very good way of comparison of fatigue lives for asynchronous and synchronous harmonic loadings. It is also very promising as a method of fatigue life prediction for asynchronous loadings.

References

Anes, V., Reis, L., Li, B. and de Freitas, M. (2014) New cycle counting method for multiaxial fatigue. International Journal of Fatigue, 67, pp. 78-94.

Böhm, M. and Niesłony, A. (2015) Strain-based multiaxial fatigue life evaluation using spectral method. Procedia Engineering, 101, pp. 52-60.

Carpinteri, A., Kurek, M., Łagoda, T. and Vantadori, S. (2016) Estimation of fatigue life under multiaxial loading by varying the critical plane orientation. International Journal of Fatigue (in press).

Dong, P., Wei, Z. and Hong, J.K. (2010) A path-dependent cycle counting method for variable-amplitude multi-axial loading. International Journal of Fatigue, 32, pp. 720-734.

Jahed, H., Varvani-Farahani, A., Noban, M. and Khalaji, I. (2007) An energy-based fatigue life assessment model for various metallic materials under proportional and non-proportional loading conditions. International Journal of Fatigue, 29, pp. 647-655.

Karolczuk, A., Kluger K. and Łagoda T. (2016) A correction in the algorithm of fatigue life calculation based on the critical plane approach. International Journal of Fatigue, 83, pp. 174-183.

Mei, J. and Dong, P. (2017) Modeling of path-dependent multi-axial fatigue damage in aluminum alloys. International Journal of Fatigue, 95, pp. 252-263.

Mei, J. and Dong, P. (2016) A new path-dependent fatigue damage model for non-proportional multi-axial loading. International Journal of Fatigue, 90, pp. 210-221.

Noban, M., Jahed, H., Winkler, S. and Ince, A. (2011) Fatigue characterization and modeling of 30CrNiMo8HH under multiaxial loading. Materials Science and Engineering A, 528, pp. 2484-2494.

Pejkowski, Ł. (2016) On the material's sensitivity to non-proportionality of fatigue loading. Archives of Civil and Mechanical Engineering (in press).

Pejkowski, Ł., Skibicki, D. and Lipski, A. (2016) Plastic strain energy of CuZn37 brass in low-cycle multiaxial fatigue regime, AIP Conference Proceedings, 1780, pp. 30004-1-30004-8.

Pejkowski, Ł. and Skibicki, D. (2014) Modification of Zenner and Liu Criterion due to Non-Proportionality of Fatigue Load by Means of MCE Approach, Key Engineering Materials 598, pp. 201-206.

Pejkowski, Ł., Skibicki, D. and Sempruch, J. (2014) High cycle fatigue behavior of austenitic steel and pure copper under uniaxial, proportional and non-proportional loading, Strojniski Vestnik - Journal of Mechanical Engineering, 60, 9, pp. 549-560.

Pejkowski, Ł., Skibicki, D. and Sempruch, J. (2012) Steel X2CrNiMo17-12-2 Testing for Uniaxial, Proportional and Non-Proportional Loads as Delivered and in the Annealed Condition. Materials Science Forum, 726, pp. 171-180.

Skibicki, D., Sempruch, J. and Pejkowski, Ł. (2014) Model of non-proportional fatigue load in the form of block load spectrum. Materialwissenschaft und Werkstofftechnik, 45, pp. 68-78.