

## APPLICATION OF WOLFRAM MATHEMATICA PACKAGE TO CONTROL THE 6-DOF PARALLEL ROBOT

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**Abstract:** *The article presents the application of Wolfram Mathematica software for solving kinematic equations in the process of rapid control prototyping for controlling the parallel manipulator 6-DOF with six degrees of freedom, delta type, with an electric drive. The process of manipulator control required the preparation of simple and inverse kinematic algorithms of the manipulator. The algorithms were implemented in the control system.*

**Keywords:** Parallel manipulator, 6-DOF, Delta, Kinematics.

### 1. Introduction

At present, the development of robotics forces the search for new kinematic structures as well as simple and reliable control systems. Already at the stage of designing, methods Rapid Control Prototyping and Hardware-in-the-loop are used (Takosoglu, 2016). In the newly designed robots, especially in structures with parallel kinematics, it is required to solve nonlinear equations with many unknown (Zwierzchowski, 2016a). Examples are known of using the Rapid Control Prototyping technology in the process of controlling and solving kinematic equations for parallel robots 3-DOF and 6-DOF (Laski et al., 2014). At present, there are many cheap development platforms on the market, such as single-circuit microprocessor based platforms like ARM and Atmega, which can be successfully applied in uncomplicated control systems (Blasiak, M., 2016). In the case of controlling a parallel robot delta 6-DOF type it is required to solve kinematic equations in real time. With view to the above, an intermediate solution was proposed: in Wolfram Mathematica software kinematic equations are solved with the use of the function FindRoot based on the Newton – Raphson method, while the development platform with the STM32F4 microcontroller is responsible for the physical process of control (Janecki et al., 2015; Blasiak, S. et al., 2014). Such a solution allows for quick prototyping the control system with a parallel robot with significantly lower financial resources involved (Nowakowski et al., 2016). The present manipulator has six degrees of freedom. It consists of a fixed base connected with a moving platform with six arms (Laski et al., 2014). Delta type manipulators are characterized by high load carrying capacity, but small working space when compared to serial structures (Pietrala, 2016). The precursors of papers on devices with closed kinematic circuits include Gough, V. E. and Stewart, D., hence parallel manipulators are commonly referred to as Gough's – Stewart's platforms.

### 2. Manipulator structure and principle of operation

Manipulator role consists in performing the movement of the working platform in relation to its fixed base. Output elements performing the movement are modelling servo-drives performing rotary movement and directly driving the arms. The arms are connected with the moving platform with ball-and-socket

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joints. The movement of the platform is the resultant displacement of all six drives of the manipulator. Due to the problem of controlling the working platform, manual control of the manipulator, consisting in the change of angular location of each drive independently of the others, known from open kinematic chain manipulators, is in this structure very difficult or even impossible. Each of manipulator arms has six degrees of freedom: one class 5 kinematic pair and two class 3 kinematic pairs (joint rotation in longitudinal axis of the upper part of the arm in both ball-and-socket joints). The solid model of the manipulator created with the use of the 3D CAD software and the general view of the manipulator were shown in Fig. 1.

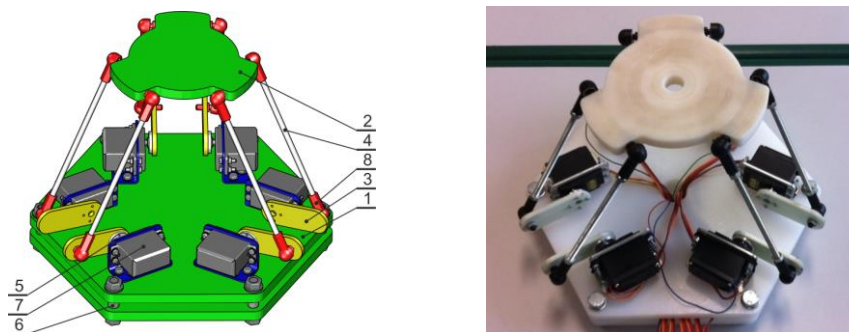


Fig. 1: Mechanism . Solid model: 1 – base plate, 2 – moving platform, 3 – lower part of the arm, 4 – upper part of the arm, 5 – drive fixing, 6 – spacing sleeve, 7 – servo-drive, 8 – ball-and-socket joint.

Due to the complicated manipulator structure, in the control process it is necessary to solve a simple and inverse kinematic equation. The simple kinematic equation consists in finding the coordinates of location and orientation of the platform knowing joint coordinates of individual arms. The inverse kinematic equation consists in finding joint coordinates of individual arms knowing the coordinates of location and orientation of the working platform. Joint coordinates have been designated as:  $\theta_{1,1}, \theta_{1,2}, \dots, \theta_{6,3}$ , where  $\theta_{1,1}, \theta_{2,1}, \theta_{3,1}, \theta_{4,1}, \theta_{5,1}, \theta_{6,1}$  are angles of rotation of drives axes. Cartesian coordinates of the location and orientation of the platform were marked as:  $P_x, P_y, P_z, M$ , where  $P_x, P_y, P_z$  are the coordinates of the centre of the working platform in the base system of coordinates  $U_0$ , and  $M$  is the matrix of nine cosines defining the orientation of the system  $UP_{i,1}$  in the base system of coordinates  $U_0$ . In order to solve the inverse kinematic equation of the manipulator, the following assumptions were adopted: the symbols used in introduced dependences shall mean –  $P_x, P_y, P_z$  - Cartesian coordinates of the set location of the manipulator,  $\alpha_{x_x}, \alpha_{x_y}, \alpha_{x_z}, \alpha_{y_x}, \alpha_{y_z}$  - direction cosines defining the set orientation of the working platforms,  $U_0$  - the base system of coordinates connected with the plate,  $Wsp_{i,x}, Wsp_{i,y}, Wsp_{i,z}$  - coordinates  $x, y, z$  of  $i$  ball-and-socket joint in the base system of coordinates  $U_0$ ,  $UP_{i,x}$  - the system of coordinates with number  $x$  connected with  $i$  ball-and-socket joint of the platform,  $UR_{i,x}$  - the system of coordinates with number  $x$  connected with  $i$  arm,  $\zeta_i, \xi_i, R, r, h, l_1, l_2$  - geometrical dimensions of individual manipulator elements

- the basic Cartesian coordinate system  $U_0$ , was connected with manipulator base. Plane  $X_0Y_0$  of that system overlaps with the surface of upper plate of the base. The beginning of the system lies in the points of crossing of base diagonals. Axis  $Z_0$  points towards the platform, and axis  $X_0$  lies on the plane which is symmetrical for one arm pair and it points towards the arms,
- the manipulator moving platform was connected with the Cartesian coordinate system  $UP_{i,1}$ , identical for all joints. That system is orientated according to the following assumptions: plane  $X_{i,1}Y_{i,1}$  lies on the plane passing through centres of platform joints, the beginning of the system of coordinates lies in the point of crossing of the platform diagonals, axis  $Z_{i,1}$  points in the opposite direction than the direction of the base, axis  $X_{i,1}$  lies on the plane symmetrical for one arm pair and it points towards the arms,
- the coordinates of platform position are the coordinates of the beginning of system  $UP_{i,1}$  in system  $U_0$ , hence platform shifts are done in relation to axis  $X_0, Y_0, Z_0$ . Coordinate  $P_x$  corresponds to the shift

along axis  $X_0$ , coordinate  $P_y$  corresponds to the shift along axis  $Y_0$ , and the coordinate  $P_z$  corresponds to the shift along axis  $Z_0$ ,

- orientation of the working platform is the orientation of system  $UP_{i,1}$  in the base system  $U_0$ . That orientation is defined by the matrix of nine direction cosines defining the position of axes of the system of coordinates  $UP_{i,1}$  in relation to the axes of the system  $U_0$ . Because both systems are dextral systems of Cartesian coordinates, five out of nine direction cosines fully define the orientation of the platform.

Solving the inverse kinematic equation of the manipulator was done in two stages:

- the first stage consists in finding the coordinates of points lying in centres of ball-and-socket joints of the platform, in relation to system  $U_0$ , knowing the set position and orientation of the platform,
- the second stage consists in finding joint variables of each manipulator arm, knowing the coordinates of ball-and-socket joints connecting an arm with the platform enumerated in the first stage.

In the first stage the manipulator base, together with the platform, should be treated as the manipulator with an open kinematic chain with six degrees of freedom. In such a structure, the platform is connected with six class 5 kinematic pairs, three of which are progress pairs and another three are rotation pairs. In such a manipulator, the simple kinematic equation should be solved for each platform joint separately, with the consideration that the positions of platform joints in relation to the platform centre may differ from one another. They are however fixed and they appear from geometrical conditions of the platform, that is why the introduced dependencies are universal and true for all joints. The problem posed in such a way was solved by using the Denavit – Hartenberg parameters (Zwierzchowski, 2016b). Kinematic equations have been determined for the structure being analyzed (1). This equations take into account the relationship tying the arms to the work platform.

$$\begin{aligned}
 f_{i,x}(\theta_{i,1}, \theta_{i,2}, \theta_{i,3}) &= -Wsp_{i,x} + R \cos \xi_i + \cos(\xi_i \pm \xi_0)(l_1 \cos \theta_{i,1} + l_2 \cos(\theta_{i,1} + \theta_{i,2}) \cos \theta_{i,3}) + \\
 &\quad - l_2 \sin(\xi_i \pm \xi_0) \sin \theta_{i,3} = 0 \\
 f_{i,y}(\theta_{i,1}, \theta_{i,2}, \theta_{i,3}) &= -Wsp_{i,y} + R \sin \xi_i + \sin(\xi_i \pm \xi_0)(l_1 \cos \theta_{i,1} + l_2 \cos(\theta_{i,1} + \theta_{i,2}) \cos \theta_{i,3}) + \\
 &\quad + l_2 \cos(\xi_i \pm \xi_0) \sin \theta_{i,3} = 0 \\
 f_{i,z}(\theta_{i,1}, \theta_{i,2}, \theta_{i,3}) &= -Wsp_{i,z} + h + l_1 \sin \theta_{i,1} + l_2 \sin(\theta_{i,1} + \theta_{i,2}) \cos \theta_{i,3} = 0
 \end{aligned} \tag{1}$$

Variables  $\theta_{i,1}, \theta_{i,2}, \theta_{i,3}$  are joint variables of individual arms, where variable  $\theta_{i,1}$  is the angle of rotation of the drive shaft of  $i$  arm. For solving the systems of equations, the function *FindRoot* of the Mathematica software was used which is based on the Newton – Raphson method which is described by the formula:

$$x_{k+1} = x_k - (F'(x_k))^{-1} \cdot F(x_k) \tag{2}$$

where:  $k$  – iteration number,  $x_k$  – approximation of the sought value calculated for a given iteration,  $F(x_k)$  – value of the function for a given approximation,  $F'(x_k)$  – value of matrices derived from partial functions  $F$  for a given approximation.

### 3. Manipulator control system

For constructing the manipulator control system a PC was used with the installed Wolfram Mathematica software, and the development platform with the built-in microcontroller STM32F103ZET6. The control system has 12-bit A/C transmitters and USART interface enabling communication in standard RS232. Communication interface USB-RS232 was used for sending information between the computer and the microcontroller. DC servo drives were used in the manipulator. These servos are equipped with potentiometric position sensors modified to access signals with the current angular position of the drive. Fig. 2 shows the general view of the manipulator connected to the control system. Then, also from the level of Mathematica software, the inverse kinematic equation was solved for the set trajectory. For that, for each manipulator arm the system of three nonlinear equations with three unknown was solved. For solving the function *flin*[ $\theta$ ] was made which uses the built-in function *FindRoot*[ $\theta$ ] based on the Newton-Raphson method. The function requires providing the initial values vector. Because the system of three equations with three unknown has more than one solution, the initial values vector must be close to the sought solution. Via the interface USB-RS232 the calculated joint variables were sent to the

microcontroller. In order to control the serial port from the level of Mathematica software, an additional library was downloaded from the website of the Wolfram company. The library contains two files: SerialIO.exe and init.m. Next, the downloaded package was started, the serial port opened and data transfer speed set. In the next step, the joint variables were calculated, converted to text and sent to the microcontroller. Suitable control signals for the drives were generated. Then, with the use of A/C transmitters, measurements of signals from angular location sensors were made. Measurement data was sent to the Mathematica software with the use of USB-RS232 interface.

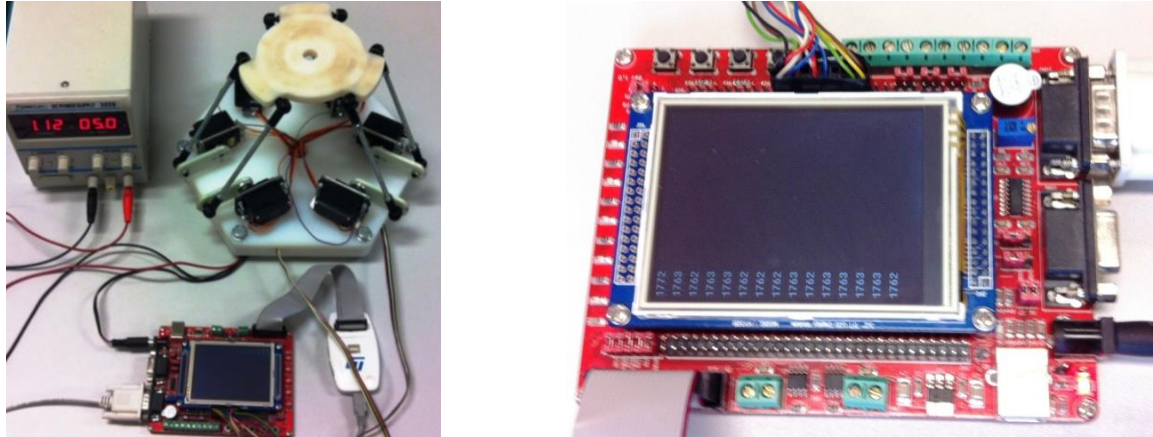


Fig. 2: General view of the manipulator with the control system microcontroller STM32F103ZET6.

#### 4. Concluding remarks

The process of controlling parallel manipulators requires solving simple and inverse kinematic equations. The use of the Mathematica software allows to simplify solving the simple and inverse kinematic equations to a great degree. Performing matrix operations enables a quick construction of the kinematic model of the manipulator with the use of in-built functions solving nonlinear systems of equations, what in turn enables determining joint variables of the manipulator. Moreover, the Mathematica software includes functions controlling the serial port of a PC or its virtual equivalent. Thanks to that, communication with most microcontrollers available on the market is possible. Such type of manipulator may be applied for instance in the process of positioning cameras, palletisation. It may be used as a machining device controlled numerically. Solving a simple kinematic equation allows to use the manipulator as a device setting the position and orientation (a haptic device).

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