23rd International Conference
ENGINEERING MECHANICS 2017
Svratka, Czech Republic, 15 – 18 May 2017

# MATHEMATICAL MODELLING OF ROTOR SYSTEMS WITH JOURNAL BEARINGS IN LIMIT CASES

**P. Polach[*], M. Hajžman[**], M. Byrtus[***], Š. Dyk[****], L. Smolík[*****]**

**Abstract:** *Dynamics of rotating systems involves behaviour and diagnostics of rotating structures. When hydrodynamic journal bearings are used to support a rotor the rotor-bearing system becomes a complex dynamic system that may exhibit serious fluid film instabilities. The understanding of the behaviour of a fluid film bearing closely before, during and after the rotor instability origin and growth is the main motivation for the complex research of local and global dynamics of a rotor-bearing system. Deep knowledge of the relations between local fluid film dynamics and dynamic response of rotating systems during instabilities can help to improve design of many modern rotating machines.*

**Keywords: Rotor dynamics, Reynolds equation, Fluid film bearings.**

## 1. Introduction

Dynamics of rotating systems supported by journal bearings is an interesting topic of computational mechanics. When hydrodynamic journal bearings are used to support a rotor the rotor-bearing system becomes a complex dynamic system that may exhibit serious fluid film instabilities related to various limit cases, which are the main subject of this paper. The understanding of the fluid film bearing behaviour closely before, during and after the rotor instability origin and growth is the main motivation for the complex research of local and global dynamics of a rotor-bearing system.

In 1925 Newkirk and Taylor discovered that a journal bearing can induce unstable vibrations of a supported rotor. This instability – commonly called oil whirl (Hori, 1959) – remains troublesome and of great concern. Lund (1962) investigated bifurcations analytically within the scope of the Hopf bifurcation theory and introduced the method of multiple scales. Later Meyers (1986) applied the bifurcation theory to oil whirl employing numerical methods. Recent development in numerical methods for nonlinear models and numerical continuation methods allowed studying the oil-induced instabilities and resulting bifurcations even deeper. Castro et al. (2008) implemented successfully nonlinear hydrodynamic forces and predicted oil whirl/whip for a real vertical power plant and a horizontal test rig. Boyaci et al. (2010) used a numerical continuation method to detect bifurcations of stationary and periodic solutions for a flexible rotor supported by two identical journal bearings. Adiletta et al. (2011) and Laha and Kakoty (2011) published the work similar to Boyaci's. Adiletta et al. (2011) studied a rigid rotor supported by two-lobe journal bearings, whereas Laha and Kakoty (2011) studied a flexible rotor supported by porous hydrodynamic journal bearings. Mishra (2012) investigated whirl ratio for an adiabatic solution of a bearing with non-Newtonian fluid. Amamou and Chouchane (2014) discussed the stability issues including hysteresis and jump phenomena of long journal bearings utilizing nonlinear model and continuation methods. Yang et al. (2014) proposed second-order nonlinear stiffness and damping

[*] Dr. Ing. Pavel Polach: New Technologies for the Information Society, European Centre of Excellence, University of West Bohemia, Univerzitní 8; 306 14, Pilsen; CZ, ppolach@ntis.zcu.cz

[**] Ing. Michal Hajžman, PhD.: New Technologies for the Information Society, European Centre of Excellence, University of West Bohemia, Univerzitní 8; 306 14, Pilsen; CZ, mhajzman@kme.zcu.cz

[***] Ing. Miroslav Byrtus, PhD.: New Technologies for the Information Society, European Centre of Excellence, University of West Bohemia, Univerzitní 8; 306 14, Pilsen; CZ, mbyrtus@rice.zcu.cz

[****] Ing. Štěpán Dyk: New Technologies for the Information Society, European Centre of Excellence, University of West Bohemia, Univerzitní 8; 306 14, Pilsen; CZ, sdyk@ntis.zcu.cz

[*****] Ing. Luboš Smolík: New Technologies for the Information Society, European Centre of Excellence, University of West Bohemia, Univerzitní 8; 306 14, Pilsen; CZ, carlist@ntis.zcu.cz

coefficients. The proposed method can be used for studying the nonlinear behaviour of the oil film instead of the continuation methods.

This paper deals with the summary of computational modelling methods related to the rotor dynamics with fluid film bearings. Basic steps of the methodology are explained and the consideration of various effects close to the limit behaviour is introduced.

## 2. Equations of motion of rotating systems

There are several common methods for the modelling and dynamical analysis of rotating systems. Two of them are mentioned in this paper.

### 2.1. Finite element modelling

A standard possibility of the dynamical modelling of rotating bodies is based on the finite element method considering one-dimensional Euler-Bernoulli or Timoshenko beams. The models respect continuous mass of rotating shafts and possible effects of lumped masses such as disks, gear wheels, etc. The mathematical model is derived in the form of the second order ordinary differential equation

$$\mathbf{M}_S \cdot \ddot{\mathbf{q}}_S(t) + \left(\mathbf{B}_S + \omega_0 \mathbf{G}_S\right) \cdot \dot{\mathbf{q}}_S(t) + \mathbf{K}_S \cdot \mathbf{q}_S(t) = \mathbf{f}_E + \mathbf{f}_B, \tag{1}$$

where $\mathbf{M}_S$ is the shaft mass matrix, $\mathbf{B}_S$ is the damping matrix, $\omega_0 \cdot \mathbf{G}_S$ represents gyroscopic effects, $\mathbf{K}_S$ is the shaft stiffness matrix, $\mathbf{f}_E$ is the general vector of external forces and $\mathbf{f}_B$ is the vector representing bearing forces, which will be described in next chapter. The whole mathematical model is derived in the configuration space defined by the vector of generalized coordinates of the shaft denoted as $\mathbf{q}_S(t)$.

### 2.2. Approaches based on multibody dynamics

Another possibility is the utilization of approaches based on the multibody dynamics. Vibrations and global motion of each flexible body are described by a system of differential algebraic equations (DAE) defined by Offner (2011) in the form

$$\begin{aligned}
\mathbf{M} \cdot \dot{\mathbf{v}} + \mathbf{f}^{rbAcc}(\mathbf{s}, \dot{\mathbf{s}}) &= \mathbf{f}^{gyros}(\mathbf{s}) + \mathbf{f}^{j}(\mathbf{s}, \mathbf{w}) + \mathbf{f}^{e}(\mathbf{s}) - (\alpha_1 \cdot \mathbf{M} + \alpha_2 \cdot \mathbf{K}) \cdot \mathbf{v} - \mathbf{K} \cdot \mathbf{q}, \\
\dot{\mathbf{q}} &= \mathbf{v}, \\
\dot{\mathbf{x}}_B &= \mathbf{v}_B, \\
\mathbf{S}(\boldsymbol{\theta}_B) \cdot \dot{\boldsymbol{\theta}}_B &= \boldsymbol{\omega}, \\
\boldsymbol{\theta}_B^T \cdot \boldsymbol{\theta}_B &= 1, \\
r(\mathbf{q}) &= 0,
\end{aligned} \tag{2}$$

where $\mathbf{s} = \left[\mathbf{x}_B^T, \boldsymbol{\theta}_B^T, \dot{\mathbf{x}}_B^T, \boldsymbol{\Omega}_B^T, \mathbf{q}^T, \dot{\mathbf{q}}^T\right]^T$ is the state vector, which includes vector $\mathbf{x}_B \in \Re^3$ and quaternion $\boldsymbol{\theta}_B \in \Re^4$ defining the position and the orientation of the body's relative coordinate system. Vectors $\dot{\mathbf{x}}_B \in \Re^3$ and $\boldsymbol{\Omega}_B \in \Re^3$ hold translational and rotational velocities of the body's relative system, and $\mathbf{q} \in \Re^N$ is the generalized displacement vector, which covers elastic deformations. Diagonal mass matrix **M**, symmetric stiffness matrix **K** and symmetric damping matrix $\mathbf{D} = \alpha_1 \cdot \mathbf{M} + \alpha_2 \cdot \mathbf{K}$ are constant in time due to the formulation with respect to the body's relative system. $\alpha_1$ and $\alpha_2$ are the coefficients of proportional (Rayleigh) damping.

Vectors $\mathbf{f}^{gyros}(\mathbf{s}) \in \Re^{6 \cdot N}$ and $\mathbf{f}^{rbAcc}(\mathbf{s}, \dot{\mathbf{s}}) \in \Re^{6 \cdot N}$ are forces resulting from gyroscopic effects and rigid body accelerations, $\mathbf{f}^{j}(\mathbf{s}, \mathbf{w}) \in \Re^{6 \cdot N}$ is the vector of joint and contact forces dependant on state vector **s** of the modelled body and state vectors of all coupled bodies combined in **w**, and $\mathbf{f}^{e}(\mathbf{s}) \in \Re^{6 \cdot N}$ is the vector of external loads. Further details are discussed by Offner (2011).

## 3. Modelling of journal bearings

The supports of rotating systems can be realized using journal bearings. Elasto-hydrodynamic forces acting in radial journal bearings are computed from bearing loads *W*, which can be expressed as an integral of oil pressure $p = p(x, z, t)$ over bearing surface *A* showed by Stachowiak and Batchelor (2014) in the form

$$W = -\iint\limits_{(A)} \cos x \cdot p(x,z) \mathrm{d}x\mathrm{d}z, \tag{3}$$

where *x* and *z* are circumferential and axial coordinates in a coordinate frame fixed to a bearing shell, where the origin of circumferential coordinate *x* is located on the upper point of the shell. Hydrodynamic oil pressure *p* is computed employing modified averaged Reynolds equation defined by Offner (2013) in the form

$$\frac{\partial}{\partial x}\left(\frac{\phi_x \cdot \theta \cdot h^3}{12 \cdot \eta} \cdot \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\phi_z \cdot \theta \cdot h^3}{12 \cdot \eta} \cdot \frac{\partial p}{\partial z}\right) = \frac{u_j - u_s}{2} \cdot \frac{\partial\left[\theta \cdot \left(h_T + \sigma_c \cdot \phi_s\right)\right]}{\partial x} + \frac{\partial\left(\theta \cdot h_T\right)}{\partial t}, \tag{4}$$

where $h = h(x, z, t)$ represents a nominal clearance gap between the shell and the journal, $h_T = h_T(x, z, t)$ is the total clearance gap including surface roughness, $\theta = \theta(x, z, t)$ is the fill ratio, i.e. percentage of gap *h* filled with oil, and $\eta$ is the dynamic viscosity of oil. Stoke's sticking condition in *x* direction are considered in equation (4) with boundary conditions $u_j$ and $u_s$ being circumferential velocities of the journal and the shell. Couette flow factors $\phi_x$, $\phi_z$ and shear flow factor $\phi_s$ (Prat et al., 2002) and composite roughness $\sigma_c$ (Greenwood and Tripp, 1970) are also considered. Equation (4) should be solved for pressure $p$ in lubrication regions and for $\theta$ in cavitation regions. Once the film pressure distribution is obtained, the elasto-hydrodynamic forces acting on the system components can be expressed by integrating hydrodynamic pressure $p$ over the bearing inner surface.

## 4. Consideration of limit interaction

There are certain limit cases that could be considered in the rotor-bearing interaction. Majority of such effects is related to the variations of the clearance gap and consequent changes of force effects during a bearing operation. It comprises modifications or alternative solutions of the Reynolds equation or additional contact and friction influences.

Surface roughness starts to play an important role in the system response (Lin, 2012; Bhaskar et al., 2013; Li et al., 2015). Wear occurs when contact of surfaces is direct. Wear and thus change of geometry further influence behavior of the system (Papadopoulos et al., 2008; Machado and Cavalca, 2015). Direct contact may also trigger another instability called dry whip that may lead to a machine catastrophic failure. Moreover, a rotor-to-stator rubbing changes significantly the dynamic characteristics of the system (Ma et al., 2015; Xiang et al., 2015).

## 5. Conclusions

Mathematical modelling of rotating systems supported by journal bearings is still a big issue and dynamical analysis of these systems brings interesting findings. Approaches to the modelling of rotating shafts based on the finite element method and on the multibody system dynamics are summarized in this paper. Fluid film bearing models are mainly related to the Reynolds equation of various modifications. The models considering the limit behaviour operate with surface roughness, contact interaction, friction effects, etc.

**Acknowledgement**

The paper has originated in the framework of solving the project of the Czech Science Foundation No. 17-15915S entitled "Nonlinear dynamics of rotating systems considering fluid film instabilities with the emphasis on local effects".

## References


Adiletta, G., Mancusi, E. and Strano, S. (2011) Nonlinear behaviour analysis of a rotor on two-lobe wave journal bearings. Tribology International, 44, 2, pp. 42-54.

Amamou, A. and Chouchane, M. (2014) Nonlinear stability analysis of long hydrodynamic journal bearings using numerical continuation. Mechanism and Machine Theory, 72, pp. 17-24.

Bhaskar, S.U., Hussain, M.M. and Ali, M.Y. (2013) Stability analysis on plain journal bearing with effect of surface roughness. International Journal of Scientific & Engineering Research, 4, 3, pp. 1-8.

Boyaci, A., Seeman, W. and Proppe, C. (2010) Stability and bifurcations of rotors in fluid film bearings. Proceedings in Applied Mathematics and Mechanics, 10, 1, pp. 235-236.

Cole, J.A. and Hughes, C.J. (1956) Oil flow and film extent in complete journal bearings. Proceedings of the Institution of Mechanical Engineers, 170, 1, pp. 499-510.

De Castro, H.F., Cavalca, K.L. and Nordmann, R. (2008) Whirl and whip instabilities in rotor-bearing system considering a nonlinear force model. Journal of Sound and Vibration, 317, 1-2, pp. 273-293.

Greenwood, J.A. and Tripp, J.H. (1970) The contact of two nominally flat rough surfaces. Proceedings of the Institution of Mechanical Engineers, 185, pp. 625-633.

Hori, Y. (1959) A Theory of oil whip. ASME Journal of Applied Mechanics, 26, 2, pp. 189-198.

Laha, S.K. and Kakoty, S.K. (2011) Nonlinear dynamic analysis of a flexible rotor supported on porous oil journal bearings. Communications in Nonlinear Science and Numerical Simulation, 16, 3, pp. 1617-1631.

Li, Y.-H., Yu, X.-D., Li, C. and Meng, X.-L. (2011) Study of Monitoring for Oil Film Thickness of Elastic Metallic Plastic Pad Thrust Bearing. Advanced Design and Manufacture III, 450, pp. 239-242.

Lin, J.-R. (2012) The surface roughness effects of transverse patterns on the Hopf bifurcation behaviors of short journal bearings. Industrial Lubrication and Tribology, 64, 5, pp. 265-270.

Lund, J.W. (1967) Oil whip whirl orbit of a rotor in sleeve bearings. ASME Journal of Engineering for Industry, 89, 4, pp. 813-823.

Ma, H., Zhao, Q., Zhao, X., Han, Q. and Wen, B. (2015) Dynamic characteristics analysis of a rotor-stator system under different rubbing forms. Applied Mathematical Modelling, 39, 8, pp. 2392-2408.

Machado, T.H. and Cavalca, K.L. (2015) Modeling of hydrodynamic bearing wear in rotor-bearing systems. Mechanics Research Communications, 69, pp. 15-23.

Meyers, C.J. (1986) Bifurcation theory applied to oil whirl in plain cylindrical journal bearings. ASME Journal of Applied Mechanics, 51, 2, pp. 244-250.

Newkirk, B.L. and Taylor, H. (1925) Shaft whipping due to ill action in journal bearings. General Electric Review. 28, 8, pp. 559-568.

Offner, G. (2011) Modelling of condensed flexible bodies considering non-linear inertia effects resulting from gross motions. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, 225, 3, pp. 204-219.

Offner, G. (2013) Friction power loss simulation of internal combustion engines considering mixed lubricated radial slider, axial slider and piston to liner contacts. Tribology Transactions, 56, 3, pp. 505-515.

Papadopoulos, Ch.A., Nikolakopoulos, P.G. and Gounaris, G.D. (2008) Identification of clearances and stability analysis for a rotor-journal bearing system. Mechanism and Machine Theory, 43, 4, pp. 411-426.

Prat, M., Plouraboué, F. and Letalleur, N. (2002) Averaged Reynolds equation for flows between rough surfaces in sliding motion. Transport in Porous Media, 48, 3, pp. 291-313.

Stachowiak, G.W. and Batchelor, A.W. (2014) Engineering Tribology. 4th ed. Butterworth-Heinemann, St. Louis.

Xiang, L., Hu, A., Hou, L., Xiong, Y. and Xing, J. (2016) Nonlinear coupled dynamics of an asymmetric double-disc rotor-bearing system under rub-impact and oil-film forces. Applied Mathematical Modelling, 40, 7-8, pp. 4505-4523.