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SPECIAL ELEMENT FOR NUMERICAL MODELLING OF THE ROCK BOLT REINFORCEMENT

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Abstract: *Rock bolts as construction elements are often used in underground civil engineering projects. This work deals with their mathematical modelling. Finite elements of Aydan type were used for the description of rock bolts and hexahedral quadratic finite elements were used for the description of rock massif. A new code for the computation of stiffness matrices and right hand sides of these elements was developed by us. Stresses in a rock massif around an excavation reinforced by rock bolts were computed using the mentioned code and the solver PMD (noncommercial code developed at the Institute of Thermomechanics AS CR). The results show that the use of rock bolts can reduce the area of the maximal mechanical stress in the vicinity of excavations.*

Keywords: **Rock bolt, Finite element method (FEM), Element of Aydan type, Tunnels, Stiffness matrix.**

1. Introduction

Rock bolts as reinforcing construction elements are often used in underground civil engineering projects, especially tunnels. A typical rock bolt is created by a steel bar fastened by a cement grout in a borehole (Fig. 1). The reduction of the area of maximal mechanical stress in the vicinity of excavation is a main function of rock bolts. They can also stabilize separated blocks of rock.

Nowadays it is common to use numerical modelling for designing of various types of constructions, and rock bolts are no exception. Several special finite elements for rock bolt modelling were developed. The most widely used element was presented in the paper of Aydan (1989). The so-called Aydan element consists of two groups of nodes. The first group represents a rod sub-element, which is a simple model of the steel bar. The remaining nodes are located on the interface of the cement grout and the rock massif. The connection of the bar with the surrounding rock by the cement grout is represented by the joint action of both groups of nodes. This paper is focused on a six-node type of the Aydan element with quadratic shape functions, which is used in the 3D model and which has not been presented in literature until these days. The description of this element and its application in the 3D model of an excavation reinforced by rock bolts are presented below.

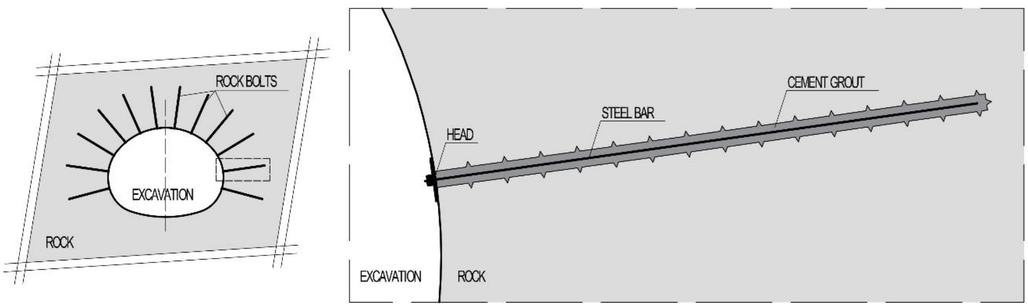

Fig. 1: Rock bolt as reinforcing construction.

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2. Classical formulation of a linear elasticity problem

Differential equations describe real physical processes inside the material. Therefore the classical formulation of the linear elasticity problem, described for instance in Brdička (1959), is mentioned first.

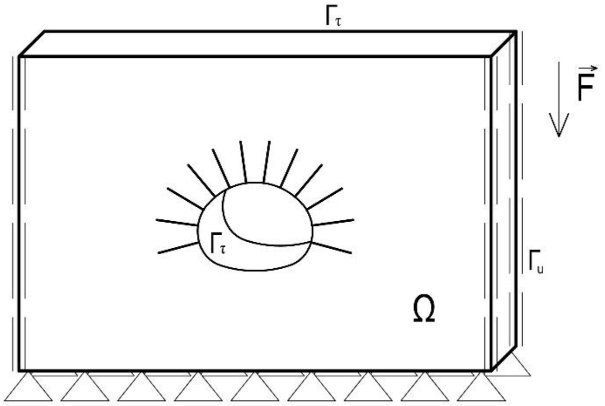

Fig. 2: Classical formulation of the linear elasticity problem.

Linear elastic body, that occupies the domain Ω (Fig. 2), is considered. We look for the vector of displacement $\boldsymbol{u} = (u_1, u_2, u_3)$ satisfying Lamé equations in the domain Ω, see Eq. (1).

$$(\lambda + \mu) \sum_{j=1}^{i} \left(\frac{\partial^2 u_j}{\partial x_i \partial x_j} \right) + \mu \Delta u_i + F_i = 0 \quad (i = 1, 2, 3), \tag{1}$$

where λ, μ are Lamé coefficients, u_i is a component of the displacement vector, x is a coordinate, F_i is a component of density of volumetric load. Two types of boundary conditions are considered,

$$u_i = u_i^0, \;\; (i = 1, 2, 3) \rightarrow \text{on } \Gamma_u, \tag{2}$$

$$\tau_{ij} v_j = T_i^0, \;\; (i = 1, 2, 3) \rightarrow \text{on } \Gamma_\tau, \tag{3}$$

where u_i is a component of the displacement vector, u^0 is a prescribed displacement, τ_{ij} is a component of the stress tensor, v_i is an outward unit normal to the boundary of Ω, T^0 is a prescribed stress vector.

We prescribed zero displacements on the bottom, on sides and on the front and back faces of the domain Ω, see Eq. (2). The zero stress vector is prescribed on the surface inside the excavation and on the top surface, see Eq. (3).

3. The finite element method

3.1. Rock bolt element of the Aydan type

The rock bolt element with quadratic shape functions has six nodes (Fig. 3). Three of them represent the steel rod (nodes 1, 2 and 3). The others are located on the interface between the cement grout and the surrounding rock. The rock bolt element is connected to elements which represent the rock massif by nodes 4, 5 and 6 (Fig. 3). The connection of the bar with the surrounding rock by the cement grout is represented by the joint action of both groups of nodes.

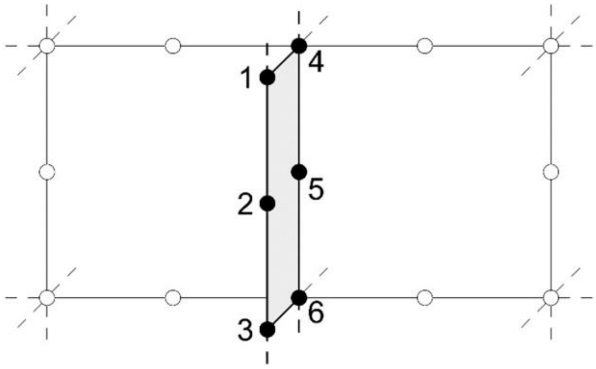

Fig. 3: Six-node rock bolt element and its connection to the finite element mesh.

Several simplifications were considered during the derivation of the stiffness matrix of the Aydan element. The steel rod and body formed by the fastening material are assumed axially symmetric and coaxial bodies. Both mentioned materials are considered homogenous, isotropic and linear elastic. Because of this assumption, dependence between stresses and deformations is described by linear Hook's law. The radius of the rock bolt is negligible with respect to nodal coordinates. Therefore, nodes 1 and 4 have identical coordinates. The same is valid for nodes 2 and 5 or 3 and 6. However, the assumption of the negligible radius cannot be applied to the process of the derivation of the stiffness matrix.

Only four types of deformations of the Aydan element are included in the computation:

a) relative longitudinal deformation of the steel bar caused by different axial displacements of nodes 1, 2 and 3,
b) relative cross shear deformation of the steel bar caused by different radial displacements of nodes 1, 2 and 3,
c) relative longitudinal shear deformation of fastening material caused by different axial displacements of nodes 1, 2 and 3 with respect to nodes 4, 5 and 6,
d) relative cross deformation of fastening material caused by different radial displacements of nodes 1, 2 and 3 with respect to nodes 4, 5 and 6.

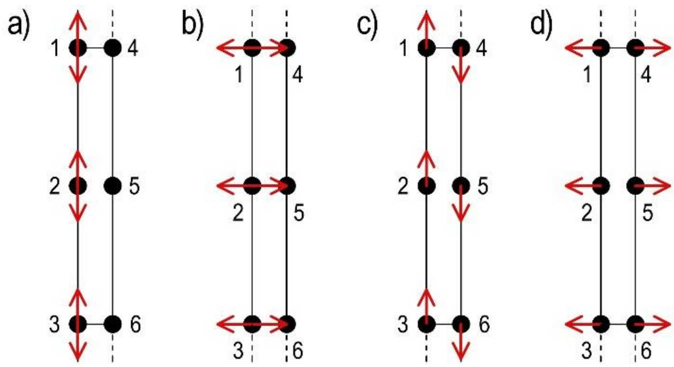

Fig. 4: Types of deformations of the Aydan element.

The stiffness matrix of the Aydan element is expressed as a volume integral. We consider constant values of displacement across the cross section of the rock bolt. Therefore, we are able to reduce the volume integral to the one-dimensional integral with an integration area of the length of the rock bolt element. Three-point Gaussian numerical integration was used for calculation of this integral.

3.2. Hexahedron – rock element for 3D model

In the 3D model the rock massif is represented by hexahedral elements with 20 nodes and quadratic shape functions, which are described, for instance, in Babuška and Szabo (1976). Eight nodes are located in vertices, remaining twelve are located in the centers of edges (Fig. 5).

The stiffness matrix and the right hand side of the hexahedral element is expressed as volume integrals. Gaussian numerical integration was used for calculating of these integrals. 27 Gauss points are needed for each element. Weights of these points were defined by the modification of the method in a one dimension by Fubini's theorem. This method is described in the work of Babuška and Szabo (1976).

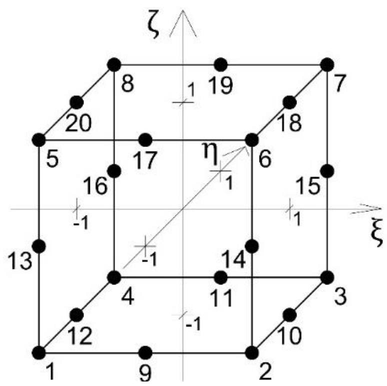

Fig. 5: Nodes of hexahedral element.

4. Model of excavation reinforced by rock bolts

Geometry of the model corresponds to the characteristic cross section of the Brusnice tunnel, which is a part of the Blanka tunnel complex. All data necessary for the creation of the model were taken from the dissertation of Nosek (2015). The excavation height is about 12.8 m and the width is about 16.6 m. The rock massif is formed by mildly eroded slates which are very common in the surrounding of the tunnel.

Two studies were considered. In the first one we assumed the excavation without any rock bolts. They were added only to the second calculation. We are able to evaluate the influence of rock bolts by a comparison of results of both studies.

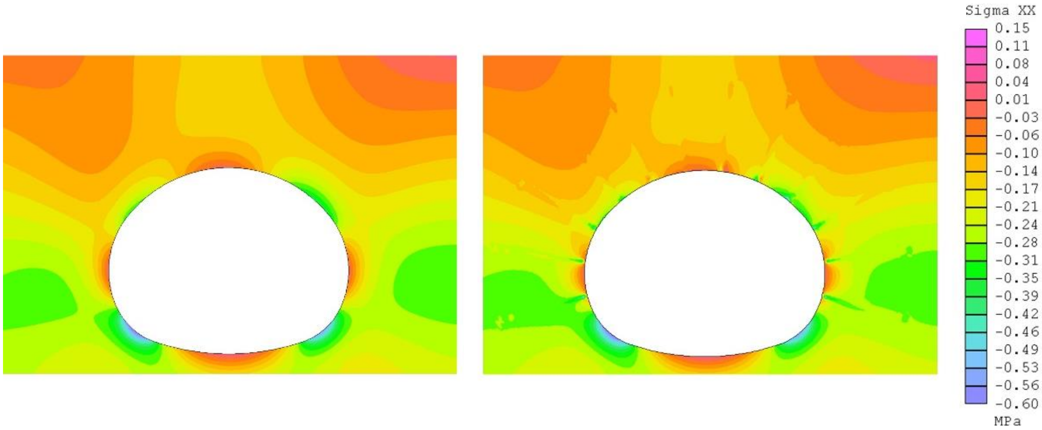

Fig. 6: Stress σ_{xx} in the surrounding of the excavation without rock bolts (left) and with rock bolts (right).

5. Conclusions

The results show that use of rockbolts can reduce areas of the maximal mechanical stress in the vicinity of excavations. In general, rock bolts help to create a rock arch by increasing normal stress in the radial direction. Described influence corresponds to theoretical knowledge of the function of rock bolts.

It is possible to combine special rock bolt elements with other types of elements with appropriate shape functions. This is the way how to create complex numerical models of reinforced excavations. Due to the relative simplicity of the described rock bolt element it is quite easy to create its various modifications with various shape functions. These modifications could be used in both 2D and 3D models. These modifications were described by Aydan (1989), Chao (1998) or Runt (2014, 2016).

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