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EFFECT OF CRACKS CLOSURE IN PLATES AND SHELLS UNDER COMBINED TENSION AND BENDING

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Abstract: *The stress-strain state and limiting equilibrium of cracked plates and shells in combined tension and bending have been studied. The effect of crack closure caused by bending strains is taken into account according to the model of contact of crack lips along a line. The diagrams of ultimate loads for any ratio of tensile and bending loads using the energy fracture criterion have been built.*

Keywords: Plate, Shell, Crack closure, Tension, Bending, Strength.

1. Introduction

The prediction of failure of thin-walled constructional elements with damages is incorrect without the analysis of contact interaction of crack faces in bending of plates and shells. In the present work we describe the methods and summarize the author's results of investigations of the stress-strain state and limiting equilibrium of cracked plates and shells loaded by simultaneous bending and tension (Shatskyi, 1989, 1995, 2015; Shatskyi et al., 2004; Shatskyi et al., 2005). The crack closure caused by bending strains is taken into account according to the model of contact of crack lips (Shatskyi, 1988, 1998, 2001; Zozulya, 1991; Young et al., 1992; Khludnev et al., 2000; Shatskyi et al., 2002; Liu et al., 2004; Dovbnya et al., 2014). This approach enables one to avoid contradictions caused by the mutual penetration of the opposite edges of cracks in the zone of compression stresses even within the framework of the classical theory of bending of shells.

2. Formulation of problem

We consider an isotropic plate $(x, y, z) \in \mathrm{R}^2 \times [-h, h]$ weakened by a through rectilinear crack of length $2l$ located along the line L and oriented along x-axis. The crack edges are subjected to the action of bending moments m and membrane forces n with equal values and opposite directions. The remaining surfaces of the plate, including points at infinity, are free of load. We study the influence of crack closure on the stressed state and limiting equilibrium of the plate.

We describe the edges contact within the framework of the classical Kirchhoff theory by using the model of contact along a line that is proposed in (Shatskyi, 1988, 2001). The mixed boundary problem for pair of biharmonic equations for the generalized plane strain state and the bending of plate has the form:

$$\Delta\Delta\varphi = 0, \quad \Delta\Delta w = 0, \quad (x, y) \in \mathbf{R}^2 \setminus L, \tag{1}$$

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$$N_y = -n, \quad M_y = -m, \quad [u_y] - h\,|[\theta_y]| > 0, \quad x \in L_1, \tag{2}$$

$$[u_y] = h\,|[\theta_y]| > 0, \; M_y + m = h(N_y + n)\,\mathrm{sgn}[\theta_y], \; N_y + n \le 0, \quad x \in L_2, \tag{3}$$

$$[u_y] = 0, \; [\theta_y] = 0, \; N_y + n \pm (M_y + m)/h \le 0, \; x \in L_3, \tag{4}$$

$$N_{xy} = 0, \quad Q_y^* = 0, \quad x \in L = L_1 \cup L_2 \cup L_3, \tag{5}$$

$$[N_y] = 0, \; [M_y] = 0, \quad [N_{xy}] = 0, \; [Q_y^*] = 0, \tag{6}$$

$$N_x = N_{xy} = N_y = 0, \quad M_x = M_{xy} = M_y = 0, \quad (x, y) \to \infty. \tag{7}$$

Here φ is the function of stresses, w is the deflection of the plate, $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $[u_y]$ is a crack opening displacement in the base surface of the shell, $[\theta_y]$ is the jump of the angle of rotation of the normal, N_x, N_{xy} and N_y are membrane forces, M_x, M_{xy} and M_y are moments, Q_y^* are generalized transverse forces, $L = (-l, l) = L_1 \cup L_2 \cup L_3$ is the crack contour, L_1 is the line of contact of the crack lips, and L_2 is an open section, L_3 is the section where the crack is completely closed. The points separating the domains L_1, L_2 and L_3 are a priori unknown and should be found in the course of solution of the problem.

The theoretical questions of existence, uniqueness and smoothness of solutions of such problems in Sobolev spaces have been investigated by means of the theory of variational inequalities (Khludnev and Kovtunenko, 2000).

3. Integral equations

We use the forces and moments integral expressions via the derivatives of the jump functions:

$$N_y(x, 0) = \frac{B}{4\pi}\int_L K_{11}(\xi, x)[u_y]'(\xi)d\xi, \quad M_y(x, 0) = -\frac{D}{4\pi}\int_L K_{33}(\xi, x)[\theta_y]'(\xi)d\xi. \tag{8}$$

Here $B = 2Eh$, $D = 2Eh^3/(3(1-\nu^2))$, E and ν are, respectively, the Young modulus and Poisson's ratio of the material of the plate. The kernels in integrals (8) are expressed via the fundamental solutions for the biharmonic equations.

As a result, problem (1) – (7) is reduced to a system of singular integral equations with constraints in the form of inequalities:

$$\frac{B}{4\pi}\int_L K_{11}(\xi, x)[u_y]'(\xi)d\xi = -n, \quad \frac{D}{4\pi}\int_L K_{33}(\xi, x)[\theta_y]'(\xi)d\xi = m,$$

$$[u_y](x) - h[\theta_y](x)\,\mathrm{sgn}[\theta_y](x) > 0, \quad x \in L_1;$$

$$-\frac{D}{4\pi}\int_L K_{33}(\xi, x)[\theta_y]'(\xi)d\xi - \frac{Bh}{4\pi}\mathrm{sgn}[\theta_y](x)\int_L K_{11}(\xi, x)[u_y]'(\xi)d\xi = hn\,\mathrm{sgn}[\theta_y](x) - m,$$

$$n + \frac{B}{4\pi}\int_L K_{11}(\xi, x)[u_y]'(\xi)d\xi \le 0, \quad x \in L_2;$$

$$[u_y] = 0, \; [\theta_y] = 0,$$

$$n + \int_L K_{11}(\xi, x)[u_y]'(\xi)d\xi \pm \left(m - \int_L K_{33}(\xi, x)[\theta_y]'(\xi)d\xi \right)/h \le 0, \quad x \in L_3. \tag{9}$$

If we consider the problem for cracked shell, then relations (1) should be replaced by shallow shells theory equations. In this case the kernels of the integral equations are determining by the shape and curvature of shell and by the crack orientation.

4. Fracture criterion and analysis of results

The state of limiting equilibrium of plates and shells with defects under combined tension and bending is specified by the condition of equality of the flow of energy into the crack tip to its ultimate value:

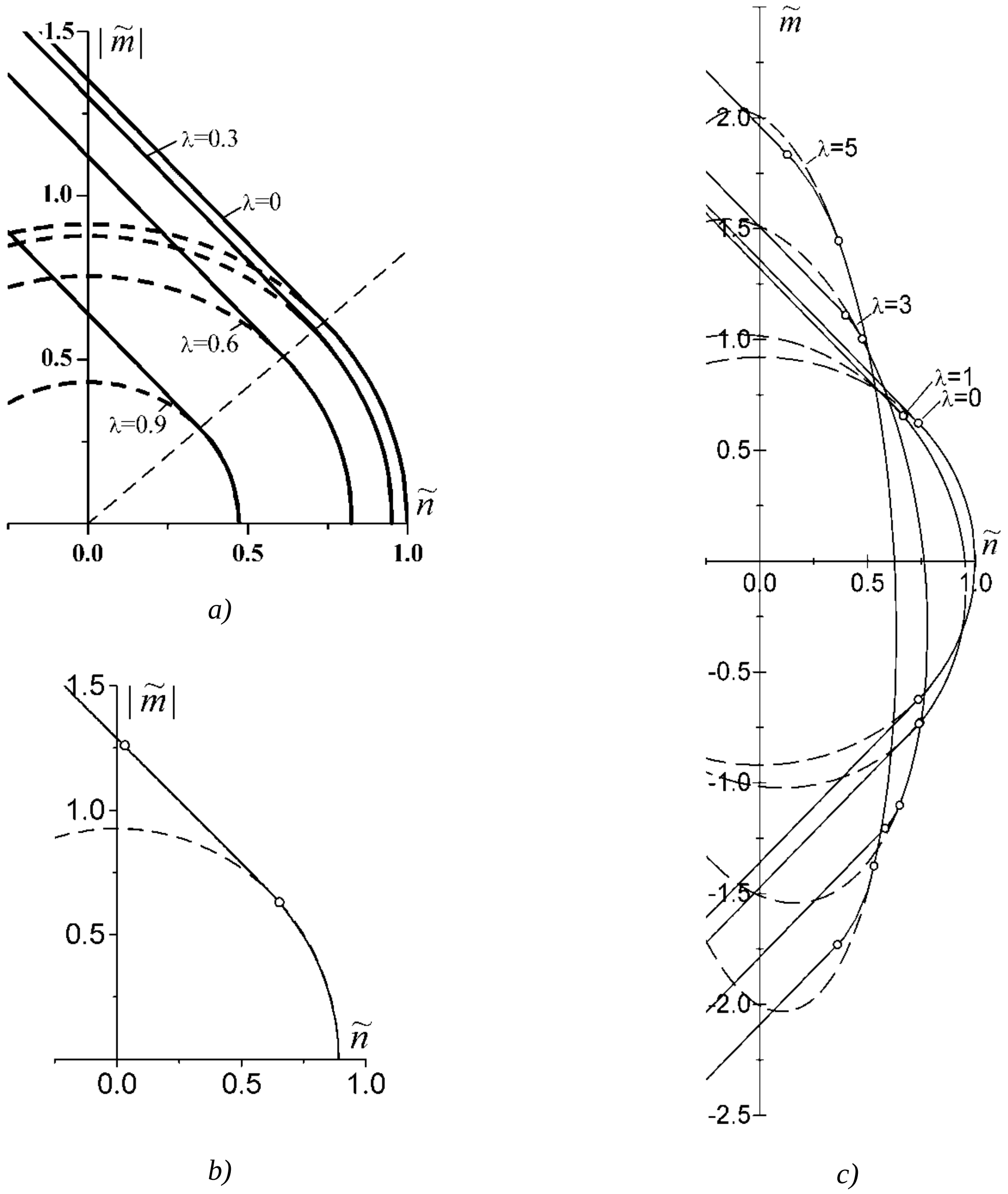

Fig. 1: Diagrams of ultimate loads: a) for plate with periodic system of collinear cracks ($\lambda = 2l/d$, d is distance between defects); b) for semi-infinite plate with edge crack of length l ; c) for cylindrical shells with circumferential crack ($\lambda = \sqrt[4]{3(1-\nu^2)}\, l/\sqrt{Rh}$, R is shell radius); the solid and dashed lines correspond to crack closure and no closure respectively; $\tilde{n} = n/n^0$, $\tilde{m} = m/(hn^0)$ and $n^0 = 2h\sqrt{2E\gamma_*/(\pi l)}$ is the Griffith force for an infinite cracked plate.

$$\frac{\pi}{4h^2 E}\left\{K_N^2 + \frac{3(1+\nu)}{3+\nu}\left(K_M/h\right)^2\right\} = 2\gamma_*,$$

where K_N and K_M are the intensity factors of forces and moments:

$$K_N = \mp \frac{B}{4\sqrt{l}} \lim_{x\to\pm l} \sqrt{l^2 - x^2}\,[u_y]'(x)\,,\ K_M = \pm \frac{D(3-2\nu-\nu^2)}{4\sqrt{l}} \lim_{x\to\pm l} \sqrt{l^2 - x^2}\,[\theta_y]'(x)$$

and γ_* is the effective surface energy of the material.

The numerical solution of system of equations (9) for uniform load ($m(x) = m = \text{const}$, $n(x) = n = \text{const}$) has been built using parametric and iterative versions of quadrature method. Some examples of simulations are presented in Fig. 1.

5. Conclusions

A model of contact of crack edges along a line is developed. The model enables us to obtain in two-dimensional statement the solutions of the problems of combined tension and bending of cracked plates and shells in the absence of kinematic contradictions.

The diagrams of limiting equilibrium of plates and shells are constructed for any ratio of the parameters of loading by tension and bending. It is shown that, in general, the contact of crack lips in bending increases the level of ultimate loads, but for the shells the ranges of the parameters of combined loading in which the effect of crack closure decreases the load-carrying capacity are established.

Despite the limited capacity of the classical plates and shells bending theory, the proposed approach allowed to avoid kinematic contradictions associated with mutual penetration of opposite surfaces of cracks in compression zones.

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