23rd International Conference
ENGINEERING MECHANICS 2017
Svratka, Czech Republic, 15 – 18 May 2017

PROPOSAL OF HYPERELASTIC PROPORTIONAL DAMPING AS DISSIPATED ENERGY MODEL OF HARD RUBBERS

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Abstract: *The paper deals with a stress analysis of hard rubber under large torsion deformations. This study was motivated by effort to find the dependency the dissipated energy on the deformation energy. Based on the results of an experiment, a function of dissipation energy of hard rubbers for finite strains using the theory hyperelasticity was proposed herein analogically as a proportional damping for elastic theory. Samples of hard rubber of different hardness (EPDM, Silicone) were dynamically tested on the developed torsional test-rig at different frequencies, amplitudes. First the Mooney Rivlin model (MRM) for a shear case of loading was analytically developed and then MRM constants were attained by fitting of the MRM to the experimental torsion-deformation curve. These constants were used to obtain the deformation energy of the MRM models. The coefficients of hyperelastic proportional damping relating a dissipated energy to a strain energy were evaluated for tested rubbers on the basis of experimental results.*

Keywords: Torsion vibration, Large deformation, Hyperelasticity, Dissipated energy, Deformation energy.

1. Introduction

For rubber materials unlike conventional structural materials under dynamic loading, a nonlinear time-varying behavior occurs due to the size of straining, creep, temperature and aging (Pešek, 2008 and Pešek, 2015). Tests of rubbers with higher hardness Sh 50 – 80 (Nashif, 1985) were performed in the laboratories of IT AS CR in recent years (Šulc, 2017). The dynamic tests of hard rubbers require usually a costly long-term operation of heavy hydraulic machines. Therefore we have started to look for realization of the tests in laboratory conditions with the lighter laboratory technique. Currently we have been developing a torsional dynamic test rig for torsional straining of hard rubber samples with a circular cross-section (Šulc, 2016). The reason for torsion straining was that hard rubber materials are softer in torsion than in pressure and therefore it is easier to achieve larger strains. Furthermore at this straining the shape changes are smaller in comparison with pressure loading when so-called barreling effect arises due to incompressibility of the material.

The torsional test rig should serve to dynamic material tests of hard synthetic rubbers for determination of the thermo-viscous-elastic material characteristics under small as well as finite strains, different amplitudes, frequencies and temperatures.

This paper deals with the deformation analysis of cylindrical samples at larger shear strains (about 30 %) which led to finding the relationship between dissipated energy and strain energy at larger torsion deformations. Function of dissipation energy of hard rubbers under finite strains was proposed as a hyperelastic proportional damping similar to modeling of damping in elastic theory. So, the dissipated energy is expressed as product of a function of deformation energy, excitation frequency and coefficient of hyperelastic proportional damping. The coefficients of hyperelastic proportional damping relating a dissipated energy to a strain energy were then evaluated for tested rubbers on the basis of experimental results. Mooney_Rivlin hyperelastic model (MRM) considered as an isotropic incompressible material is used for description of the deformation energy.

As to the temperature of the material testing we choose temperature -20 °C where the effect of excitation amplitudes of the torque to the value of the loss factor is most evident. We tested two samples of EPDM rubber of different hardness (Sh 70 and Sh 85) with excitation amplitudes of torque moment in a range

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from 0.2 N.m to 9.2 N.m. Excitation frequencies were 2 Hz and 5 Hz. Stress-strain curves and deformation and dissipated energies were evaluated for different excitation torque moment amplitudes.

2. Experimental observations

The shear modulus and loss factor for each of torque harmonic excitation amplitudes were evaluated from experimental hysteresis of deformation loops. The methodology of the evaluation is in (Šulc, 2016). The loss factors for two excitation torque frequency (2 Hz, 5 Hz) in dependence on strain [%] are plotted in Fig. 1. From dependences of the loss factor on strain, it is seen that the ratio between deformation and dissipation energy for higher values of strains is almost constant, which is typical for the proportional damping model ($B = \beta .K$) for linear elasticity. Furthermore the dependences of loss factor on the shear modulus (Fig. 2) show that the loss factor and therefore the dissipated energy decrease with increasing modulus for larger deformations. It means that dissipated energy is also related to the deformation energy. Based on these observations we proposed (see below) a function of dissipation energy of hard rubbers for finite strains analogically as a proportional damping for elastic theory and extended for hyperelasticity.

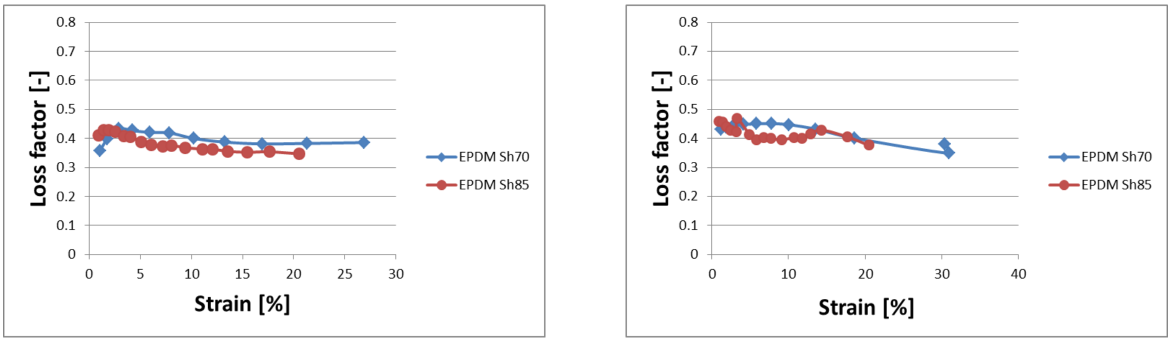

Fig. 1: The loss factor versus strain for two excitation frequencies 2 Hz (left) and 5 Hz (right) - temperature -20 °C.

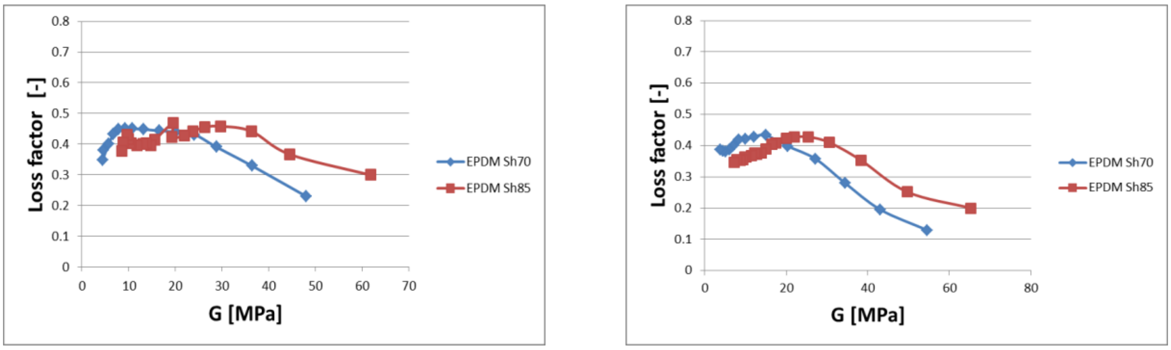

Fig. 2: The dependence the loss factor on the shear modulus at the temperature -20 °C for two excitation frequencies 2 Hz on the left and 5 Hz on the right.

3. Constants of hyperelatic model and proposal of hyperelastic proportional damping

To evaluated deformation energy analytically we needed first tune the MRM constant to experimental shear stress-strain curve. The constants of the MRM are evaluated from the experimental shear stress-strain curve using the least square method (LSM) (Šulc, 2016).

Experimental shear stress-strain curves of selected hard rubbers were obtained by our torque test rig for a temperature -20 °C. The test specimen was of cylindrical shape glued at both heads to steel consoles pins mounting in the collets. By usage of the consoles the test sample is not deformed in the vicinity of the heads due to clamping. The dimensions of the test sample of rubber were: diameter D = 0.03 m, length L = 0.095 m.

The material was isoprene butadiene rubber (EPDM) of hardness Sh 70 and Sh 85, temperature -20 °C and frequency 2 Hz of torsional loading. Experimental curves of shear stress τ verse. Strain (skew) γ, where skews maximum of about 30 % (sample Sh 70) and 20 % (sample Sh 85) are shown in Fig. 3.

Tuned constants of the three and five-parametric MRM models based on LSM method result in:

Rubber EPDM Sh 70

a) $C_{10} = -0.2172e^8, C_{11} = 0.3663e^8, C_{01} = 0.0728e^8, C_{20} = -3.4316e^8, C_{02} = 0.7382e^8$

b) $C_{10} = -1.0760e^7, C_{11} = -0.1203e^7,\ C_{01} = 0.3649e^7$

Rubber EPDM Sh 85

a) $C_{10} = -0.3067e^8, C_{11} = -0.1021e^8, C_{01} = 0.1025e^8, C_{20} = -6.2160e^8, C_{02} = 1.9206e^8$

b) $C_{10} = -1.988e^7, C_{11} = -0.4016e^7,\ C_{01} = 0.6676e^7$

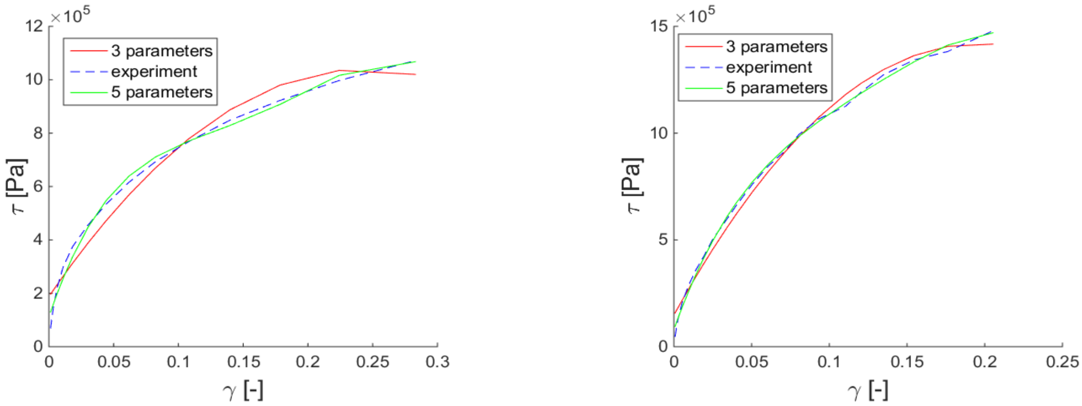

Fig. 3: Deformation curves of experiment and Mooney Rivlin models for 3 and 5 parameters EPDM Sh 70 on the left and EPDM Sh 85 on the right.

As mentioned previously based on experimental observations, dissipation energy was proposed as a hyperelastic proportional damping similar to modeling of damping in elastic theory. The dissipated energy U_{Dis} is expressed as product of the coefficient of hyperelastic proportional damping β, excitation frequency ω and deformation energy U_{Def} coming from a hyperelastic Money Rivlin model with five parameters:

$$U_{Dis} = \beta \cdot \omega \cdot U_{Def}, \tag{1}$$

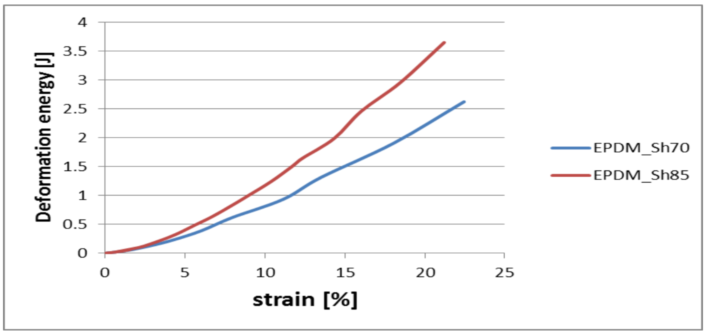

Fig. 4: The deformation energy of Money Rivlin models versus strain for EPDM Sh 70 and EPDM Sh 85 rubbers - temperature -20 °C, excitation frequency 2 Hz.

The experimental dissipation energy was obtained from the area of the hysteresis deformation loops for each case of testing torque moment. Total deformation energy was analytically evaluated from a deformation energy density u_{Def} multiplied by specific torque volume V_{red}. Deformation energy density is evaluated as integral in time over a quarter of excitation period T

$$u_{Def} = \int_0^{T/4} \tau_{\max Mooney} \cdot \dot{\gamma}_{\max} dt, \tag{2}$$

where $\tau_{\max Mooney}$ is a shear stress on surface of the cylindrical sample and $\dot{\gamma}_{\max}$ is its associated shear strain rate. Then total deformation energy is calculated from this relationship

$$u_{Def} = \int_{0}^{T/4} \tau_{\max Mooney} \cdot \dot{\gamma}_{\max} dt \,, \qquad (3)$$

The calculated total deformation energies (EPDM Sh 70 and EPDM Sh 85 rubbers) of Mooney Rivlin (5 parametric) models are in Fig. 4. It shows that the higher hardness of rubber the higher deformation energy. The hyperelastic proportional damping coefficients evaluated from expression (1) in dependence on a strain are presented for both rubbers are in Fig. 5. It shows that the coefficient is almost the same for both hardness of rubbers and it remains almost constant with a size of strain.

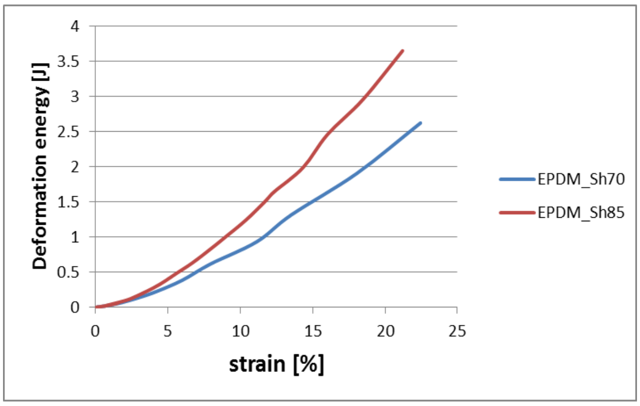

Fig. 5: The hyperelastic proportional damping coefficient versus strain for EPDM Sh 70 and EPDM Sh 85 rubbers - temperature -20 °C, excitation frequency 2 Hz.

4. Conclusions

Two EPDM hard rubbers were tested on our torque test rig for study of their material constants and damping behavior under different amplitudes and frequencies of the harmonic excitations and under finite shear deformations. The shear modulus and loss factor were first evaluated for each of the excitation settings. Based on the experimental observations, the model of hyperelastic proportional damping was proposed here and the unknown coefficients of hyperelastic proportional damping were identified for both rubbers. The results of the coefficient values show that the coefficient is almost same for both hardness of rubbers and it remains almost constant with a size of strain. Since the higher hardness of rubber the higher deformation energy the dissipation energy grows up, too, as it is related to deformation energy by the proportional damping expression. The model of dissipation will be a subject of the next research and its validation. The model will be implemented into our in-house finite element code. It enables to simulate the thermo-dynamic behavior of the rubber dampers of more complicated shapes, states of stress and finite deformations.

Acknowledgement

This work was supported by the research project of the Czech Science Foundation No. 16-04546S “Aero-elastic couplings and dynamic behaviour of rotational periodic bodies”.

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