

## THE ANALYSIS OF SELECTION OPTIMAL PARAMETERS OF PID CONTROLLERS FOR A MODIFIED ARTILLERY-MISSILE SYSTEM

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**Abstract:** *The paper presents a numerical selection method for PID controller gains used for controlling cannons azimuth and elevation angles in the modified artillery and missile system named “Wróbel II”. The selection of parameters was carried out with a method of numerical optimisation and some of the research results were presented graphically.*

**Keywords:** Controlling, PID controller, Optimisation.

### 1. Introduction

Manual tracking of a manoeuvring air target may be inaccurate in some cases. Inaccuracy may be caused by many factors including, among others, a complicated trajectory and high speeds of a target, stress caused by the aggressor’s attack and bad weather conditions during military operations. Considering that fact, it is very favourable to replace human work with automatic control systems which, on the basis of signals from a head or seeker heads (Gapiński et al., 2016), render programmed angular positions over time. In these systems, control algorithms are very important – they are responsible for efficient use of electromechanical drive units (Koruba et al., 2013 and Grzyb et al., 2016).

### 2. The system model

3D system model designed in SolidWorks is presented in Fig. 1. Basing of used construction materials it is possible to calculate masses and inertia moments of specified elements. The scheme of the system model is presented in Fig. 2.

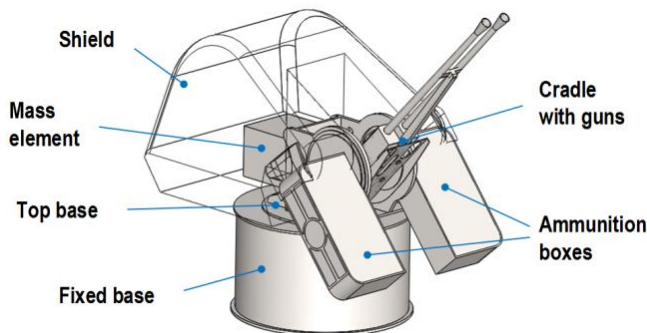


Fig. 1: 3D model of the presented artillery and missile system.

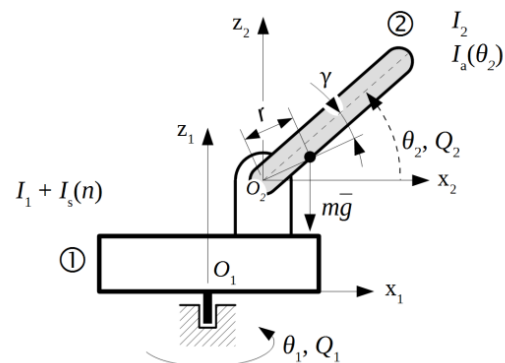


Fig. 2: The scheme of the system physical model.

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Designations presented in Fig. 2:

$\theta_1$ – azimuth angle (angle of system rotation),

$\theta_2$ – elevation angle (lifting),

$Q_i = M_i - T_i$  – generalised torque impacting on  $i$ -th element,

$M_i$  – control torque (drive) impacting on  $i$ -th element,

$T_i$  – friction torque impacting on  $i$ -th element,

$I_1$  – constant mass inertia moment of 1 element in relation to  $z_1$  axis,

$I_s(n)$  – variable mass inertia moment of 1 element in relation to  $z_1$  axis depending on a number of cartridges in  $n$  boxes,

$I_2$  – constant mass inertia moment of 2<sup>nd</sup> element in relation to  $y_2$  axis,

$I_a(\theta_2)$  – variable mass inertia moment of 2<sup>nd</sup> element in relation to  $z_1$  axis depending on elevation angle,

$m$  – mass of 2<sup>nd</sup> element,

$g$  – gravitational acceleration,

$r$  – distance from the centre of gravity of 2<sup>nd</sup> element in relation to  $y_2$  rotation axis,

$\gamma$  – angular displacement of the centre of gravity of 2<sup>nd</sup> element in relation to an axis of a gun barrel.

The Lagrange II equations were used to generate equations of the system motion. After calculating the Lagrangean and derivatives, (1) and (2) equations of generalised torques –  $Q_1$  and  $Q_2$  – were created.

$$(3a\theta_2^2 + 2b\theta_2 + c)\dot{\theta}_1\dot{\theta}_2 + (I_1 + pn + q + a\theta_2^3 + b\theta_2^2 + c\theta_2 + d)\ddot{\theta}_1 = Q_1 \quad (1)$$

$$I_2\ddot{\theta}_2 - \frac{1}{2}(3a\theta_2^2 + 2b\theta_2 + c)\dot{\theta}_1^2 + mgr \cos(\theta_2 + \gamma) = Q_2, \quad (2)$$

where:  $a, b, c, d$  – coefficients of polynomial describing the change of the inertia moment  $I_a$  in an angle function of  $\theta_2$ ;  $p$  – number of cartridges in boxes;  $n$  – coefficient depending on a cartridge mass.

Generalised torque  $Q_i$  impacting on  $i$ -th element consists of the driving torque  $M_i$  reduced by the friction torque  $T_i$  generated by movements in the element.

$$Q_i = M_i - T_i \quad (3)$$

The friction torque includes  $T_{i0}$  component which depends on velocity and  $T_{i1}$  component depending on load and impacting only during element movements (Ioannides and Guillermo, 2012).

$$T_i = T_{i0} + T_{i1} \quad (4)$$

$$T_{i0} = f_0 \cdot 10^{-7} \cdot (v \cdot n)^{2/3} d_i^3 \quad (5)$$

$$T_{i1} = u_1 \cdot f_1 \cdot P_{i0} \cdot \frac{d_i}{2}, \quad (6)$$

where:  $f_0, f_1, u_1$  – coefficients depending on bearings types;  $v$  – lubricant viscosity, mm<sup>2</sup>/s;  $n$  – rotational speed, rot/min;  $d_i$  – bearing pitch diameter, mm;  $P_{i0}$  – loading force, N

### 3. The control system structure

The adopted control system structure of angular positions of the system elements is presented in Fig. 3. The structures of the azimuth control system and the elevation control system are identical, so, as the example, the azimuth control system will be explained. The central element of the system is PID controller in a parallel, so-called, independent form.

$$u(t) = K_p e(t) + K_I \int e(t) dt + K_D \frac{d e(t)}{dt}, \quad (7)$$

where:  $K_p, K_I, K_D$  – continuous gain coefficients of proportional, integral and derivative terms,  $e(t)$  – position error;  $u(t)$  – control signal (an output of a controller) (Dębowski, 2008).

The control error  $e(t)$  is the difference between the desired signal of an angular position –  $\theta_{1zad}$  and position  $\theta_1$  detected by a displacement sensor (Stefański et al., 2014). On the basis of the error, the controller creates the control signal  $u(t)$  which is sent to the driver of the azimuth control system. The model of the drive system consists of *Rate limiter* block that is responsible for limiting torque acceleration to real values (100 N.m/s were considered), then the saturation block (maximal torque  $\pm 20$  N.m for the considered motors) and a mechanical transmission designed for increasing torque on the

motor shaft. Then signal gets into *Backlash* block causing the non-linearity in a form of backlash (0.05 °). The driving torque  $M_1$  drives the first element of the system responsible for azimuth rotation (Dziopa et al., 2012). In *The system model* block the previously presented system model was implemented. The output of the model is an angular position  $\theta_1$ . Next, the measurement is distorted with a white noise with an amplitude of  $\pm 0.01$  ° (i.e. quantisation noise from sensor).

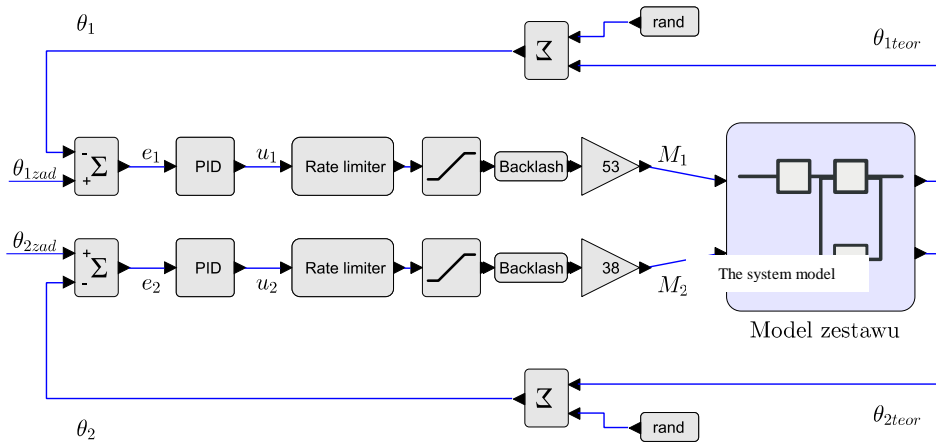


Fig. 3: General structure of the control system.

#### 4. Controller optimal parameters selection

Considering optimal adjustment some criterion was adopted. The integral of absolute error (IAE) was chosen because this index does not decrease small errors like, e.g. the integral of the squared error (ISE), which would not be favourable in accurately controlling. The procedure of optimal parameters selection used Nelder-Mead algorithm (Stachurski, 2009) in iterative simulation of the controlling system impacted by forces by using. The minimised objective function was a performance index and decision variables were PID controller gains (parameters):  $K_P$ ,  $K_I$ ,  $K_D$  (Takosoglu, 2016). Optimisation was performed subsequently for the controller of element 1 and next, for the controller of element 2. Values of the performance indexes in subsequent iterations of optimisation are presented in Fig. 4.

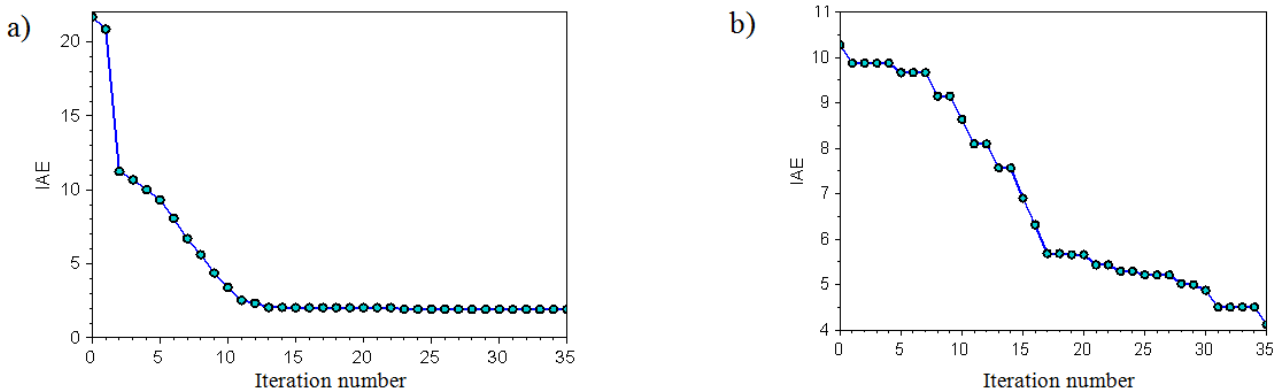


Fig. 4: Values changes of a quality index during optimisation process a) for the controller of 1 element position (azimuth), b) for the controller of 2 element position (elevation).

#### 5. The simulation of the system movement by using optimal parameters

The obtained gain coefficients for the controller of element 1 were:  $K_{P1} = 9.81$ ;  $K_{I1} = 0$ ;  $K_{D1} = 3.31$  and for the controller of element 2 were:  $K_{P2} = 9.23$ ;  $K_{I2} = 4.16$ ;  $K_{D2} = 1.31$ . Simulations of the system movements were compared to the desired signals in Fig. 5. The desired signals kept limits provided for the real system, i.e. for both elements the maximal angular acceleration did not exceed  $1.05 \text{ rad/s}^2$  ( $60 \text{ deg/s}^2$ ) and maximal speed was not higher than  $1.31 \text{ rad/s}$  ( $75 \text{ deg/s}$ ) for azimuth rotation and  $1.05 \text{ rad/s}$  ( $60 \text{ deg/s}$ ) for an elevation movement. Driving torques for specified elements are presented in Fig. 6.

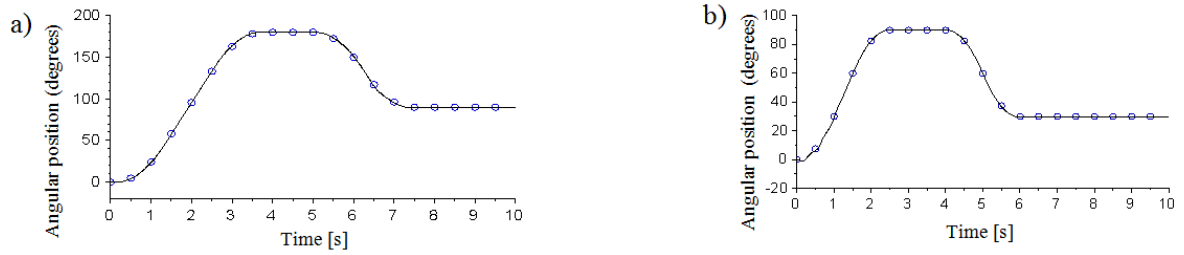


Fig. 5: Comparison of changes in desired processes  $\circ$  and processes performed  $\text{—}$  by a) element 1 (azimuth), b) element 2 (elevation).

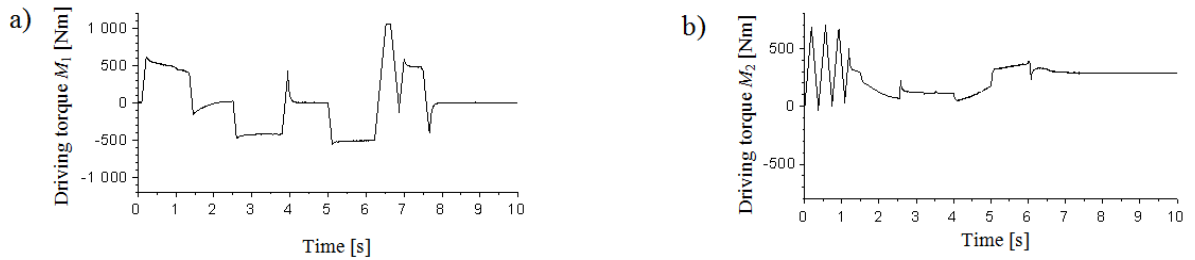


Fig. 6: Changes of driving torques of programmed movements for a) element 1, b) element 2.

## 6. Conclusions

It was ascertained that after adopting the quality and optimization criterion for controllers gain coefficients, the determined numerical parameters of controllers will allow for correct and efficient controlling both azimuth and elevation positions. The steady-state error for azimuth did not exceed  $0.02^\circ$ . For elevation the steady-state error was  $0.07^\circ$ . At the same time, it should be considered that there were backlash and noise modelled in the system. It can be stated, that tracking for desired angles is satisfactory. A further step in the work will be examination of the stability of the system for a variety of desired signals and control system robustness to change system parameters, including the variable weight of the ammunition boxes or growth moments of friction.

## References

- Dębowski, A. (2008) Automation: the basic theory. WNT, Warsaw (in Polish).
- Dziopa, Z. and Koruba, Z. (2012) Modelling and the Elements of Controlled Dynamics of the Anti-Aircraft Missile Launcher Based Onboard the Warship. Mechatronics System, Mechanics and Materials. Vol. 180, pp. 269-280.
- Gapiński, D. and Koruba, Z. (2016) Analysis of reachability areas of a manoeuvring air target by a modified maritime missile-artillery system ZU-23-2MRE. Dynamical Systems: Theoretical and Experimental Analysis, Springer Proceedings in Mathematics & Statistics, Vol. 182, pp. 125-144.
- Grzyb, M. and Stefanski, K. (2016) The use of special algorithm to control the flight of anti-aircraft missile, in: Proc. 22th Int. Conf. Eng. Mech. 2016 (eds. Zolotarev, I. and Radolf, V.), Svatka, Czech Republic, pp. 174-177.
- Ioannides, E. and Guillermo, M. (2012) Handbook of Lubrication and Tribology: Theory and Design, Second Edition, Vol. II, pp. 49-1:49-38, CRC Press Taylor & Francis Group, Boca Raton.
- Koruba, Z. and Krzysztofik, I. (2013) An algorithm for selecting optimal controls to determine the estimators of the coefficients of a mathematical model for the dynamics of a self-propelled anti-aircraft missile system. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, 227, 1, pp. 12-16.
- Stachurski, A. (2009) An introduction to optimization. Publishing House of Warsaw University of Technology, Warsaw (in Polish).
- Stefanski, K., Grzyb, M. and Nocon, L. (2014) The analysis of homing of aerial guided bomb on the ground target by means of special method of control, in: Proc. 2014 15th Int. Carpathian Control Conf. (eds. Petras, I., Podlubny, I., Kacur, J., and Farana, R.), IEEE, pp. 551-556.
- Takosoglu, J.E. (2016) Control system of delta manipulator with pneumatic artificial muscles, in: Proc. 22th Int. Conf. Eng. Mech. 2016 (eds. Zolotarev, I. and Radolf, V.), Svatka, Czech Republic, pp. 546-549.