23rd International Conference
ENGINEERING MECHANICS 2017
Svratka, Czech Republic, 15 – 18 May 2017

INFLUENCE OF LONGITUDINAL ELASTIC SUPPORT ON STABILITY OF A PARTIALLY TENSIONED COLUMN

S. Uzny*, M. Osadnik**

Abstract: *The paper presents the results of theoretical and numerical research on the stability of a partially tensioned column subjected to the Euler's load which is an external force applied between the ends of the slender system. Discrete element in a form of a translational spring which controls the longitudinal displacement was used on upper end of the system. The differential equations of motion and boundary conditions of considered column have been obtained on the basis of Hamilton's principle and Bernoulli – Euler's theory. In frame of this study the relationships between critical load and parameters such as translational spring stiffness or location of the external load were investigated.*

Keywords: Slender systems, Stability, Spring elements, Euler's load, Hamilton's principle.

1. Introduction

The aim of the stability investigations is to determine the maximum loading force (in this case the longitudinal force) at which the system will not be destroyed due to the loss of stability. In the scientific literature stability of slender flexible systems is described for both conservative and non-conservative loads. Euler's load applied to the considered system is belongs to the conservative group. Uzny et.al. (2016) first began the research on the partially tensioned slender systems subjected to Euler's load. The load was placed between the ends of the fixed-fixed column and point of force location has changed along the length of the structure. The numerical calculations have shown that the first natural vibration frequency of the studied system depends both on the point of location and magnitude of the external force. Such systems are geometrically non-linear, in which the nonlinear component of the natural frequency depends on an amplitude of the vibration. In the papers (Tomski 1985, Uzny 2011) due to the geometric non-linearity of the considered systems the solution of the boundary problem was done with small parameter method. On the basis of mathematical model authors determined the bifurcation load where the investigated systems change the rectilinear form of equilibrium into the curvilinear one. Tomski and Kukla (1989) studied slender supporting systems subjected to eccentrically applied Euler's load on both ends of the system. Additional discreet elements have great influence on critical or bifurcation load magnitude and natural vibration frequency of the systems. In the considered system the translational spring limits the axial displacement and can be adapted to the real structure to control the vibration frequency and the critical load. Discrete elements are widely used because of easy modelling of real objects. Spring elements are important for the study of vibration and stability of flexible systems. Properly chosen can affect the way of loss of stability if the slender system is subjected to compressive non-conservative load (Sundararajan 1976, Ryu et al. 2000, Kounadis 1981, 1983). Discrete components in the form of rotational springs are often used for crack modeling of flexible systems (Sokół 2014, Sokół and Uzny 2016). In the point of crack presence the reduction of local stiffness takes place thereby the discontinuity of the structure occurs, that has great influence on vibration frequency magnitude. These studies are important for the detection of cracks that may contribute to the destruction of the object.

In this work the spring can be used to model vibration isolator placed at the upper fixing of the mechanical screw mounted in the vertical lift platform. The main scope of the studies presented in this paper is to determine the critical load of partially tensioned column in relation to the point of application

* Sebastian Uzny, Prof.: Institute of Mechanics and Machine Design Foundation, Częstochowa University of Technology Dąbrowskiego 73, 42-200 Częstochowa, Poland, uzny@imipkm.pcz.pl

** Michał Osadnik, MSc.: Institute of Mechanics and Machine Design Foundation, Częstochowa University of Technology Dąbrowskiego 73, 42-200 Częstochowa, Poland, m.osadnik@imipkm.pcz.pl

of the external force for different stiffness of translational spring which is used to optimize the considered system by means of longitudinal displacement control.

2. Boundary problem

The system presented in this paper models a screw along which moves a nut loaded by external force. The considered system (column) is shown in Fig. 1. The column is subjected to Euler's load. The direction of a force is always accordant to the undeformed axis of the column. External loading force *P* was applied at the point marked with the letter *O*. Applied load causes that the lower part of the column is compressed while the upper one is tensioned. In addition, longitudinal displacement of the tensioned part of the column is limited by the translational spring of stiffness *C*. In order to formulate the boundary problem, the overall length of the system is divided into two parts of length l_1 and l_2 respectively ($l_1 + l_2 = l$). The compressed part is indicated by the index *1* and tensioned one is designated by the index *2*. The ends of the system (both on the compressed and tensioned section) are fitted in such a manner that their transversal displacements and the deflection angles are null.

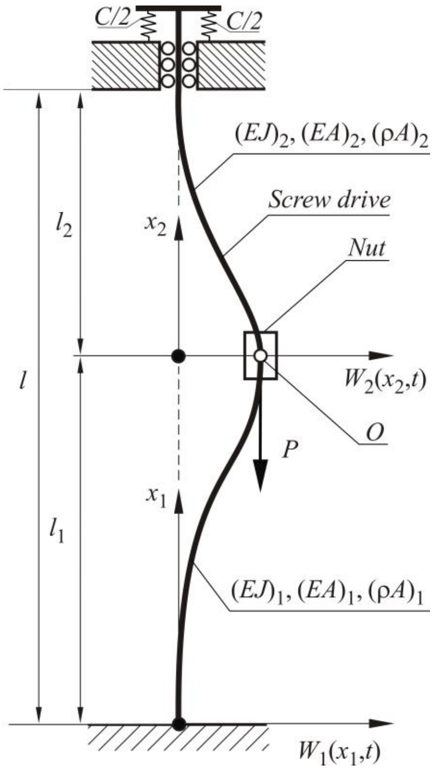

Fig. 1: Physical model of investigated system.

In this study the problem of stability is considered in order to determine the critical force.

Differential equations of transversal displacements and boundary conditions in the static case are as follows:

- differential equations of transversal and longitudinal displacements

$$W_{i0}^{IV}(x_i) + \frac{S_{i0}}{(EJ)_i} W_{i0}^{II}(x_i) = 0 \, ; \; U_{i0}(x_i) - U_{i0}(0) = -\frac{S_{i0}}{(EA)_i} x_i - \frac{1}{2}\int_0^{x_i} \left(W_{i0}^{I}(x_i)\right)^2 dx_i \qquad \text{(1a-b)}$$

- geometrical and natural boundary conditions

$$U_{10}(0) = W_{10}(0) = W_{10}^{I}(0) = W_{20}(l_2) = W_{20}^{I}(l_2) = 0 \, ; \; U_{10}(l_1) = U_{20}(0) \, ; \; W_{10}(l_1) = W_{20}(0) \, ; \; W_{10}^{I}(l_1) = W_{20}^{I}(0)$$

$$-(EJ)_1 W_{10}^{II}(l_1) + (EJ)_2 W_{20}^{II}(0) = 0 \, ; \; (EJ)_1 W_{10}^{III}(l_1) - (EJ)_2 W_{20}^{III}(0) + S_{10} W_{10}^{I}(l_1) - S_{20} W_{20}^{I}(0) = 0$$

$$S_{10} - S_{20} - P = 0 \, ; \; S_{20} - C U_{20}(l_2) = 0 \qquad \text{(2a-h)}$$

where: $W_{i0}(x_i)$, $U_{i0}(x_i)$ – transversal and longitudinal static displacements, A_i - cross-sectional area, J_i - 2nd moment of area, S_{i0} - internal force in individual rods of column in the static case, $(EJ)_i$ - bending stiffness, $(EA)_i$ - compression stiffness; $(EJ)_1 = (EJ)_2$; $(EA)_1 = (EA)_2$. The index i refers to the i-th element of the column.

Solutions of differential equations can be written as:

$$W_{10}(x_1) = A_1 \sin\left(\frac{S_{10}}{(EJ)_1} x_1\right) + B_1 \cos\left(\frac{S_{10}}{(EJ)_1} x_1\right) + C_1 x_1 + D_1 \tag{3a}$$

$$W_{20}(x_2) = A_2 \sinh\left(\left|\frac{S_{20}}{(EJ)_2}\right| x_2\right) + B_2 \cosh\left(\left|\frac{S_{20}}{(EJ)_2}\right| x_2\right) + C_2 x_2 + D_2 \tag{3b}$$

Internal forces in compressed part and tensioned one are determined with the following formulas:

$$S_{10} = P \frac{\frac{1}{Cl_2} + \frac{1}{(EA)_2}}{\frac{1}{Cl_2} + \frac{1}{(EA)_1}\frac{l_1}{l_2} + \frac{1}{(EA)_2}} \; ; \; S_{20} = S_{10} - P \tag{4a-b}$$

Substitution of the solutions (3a, 3b) into boundary conditions of transversal displacements leads to the set of equations. Determinant of the matrix of coefficients equated to zero is an equation that is used to determine the critical forces.

3. Results of numerical simulations

The results of numerical calculations are presented in the non-dimensional form by means of the following parameters:

$$\lambda_{cr} = \frac{P_{cr} l^2}{(EJ)_1} \; ; \; c = \frac{Cl}{(EA)_1} \; ; \; \zeta = \frac{l_1}{l} \tag{4a-c}$$

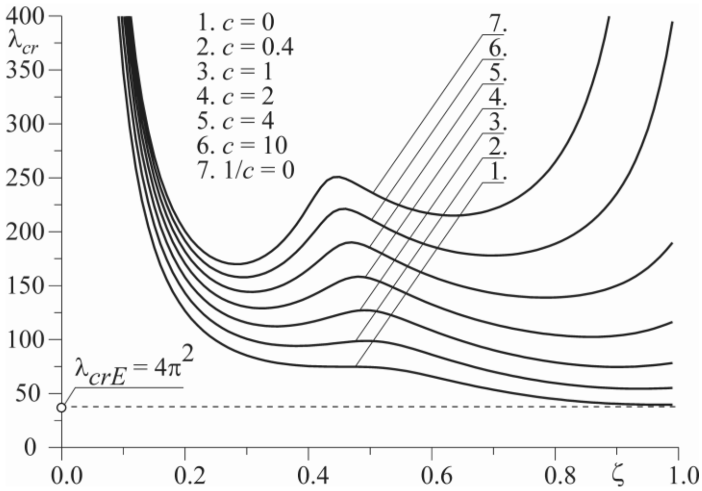

Fig. 2: The critical load λ_{cr} - point of external load application ζ relationship at different translational spring stiffness c.

Change of critical load parameter λ_{cr} depends on the ζ coefficient what is shown in Fig. 2. The numerical calculations were performed at seven different magnitudes of the stiffness of the translational spring that affects the longitudinal displacements of the upper end of the column. The influence of the translational spring stiffness parameter on critical load parameter λ_{cr} is greater at greater magnitudes of ζ coefficient. The reduction of the spring stiffness decreases the critical load, especially when external load is placed in the upper part of a considered column. At lower localized points of an application of an external load (ζ = 0.1 – 0.2) the critical load decreases very rapidly. Fig. 2 shows very clearly that when the point of application of the external load is closer to the end of the column at which the spring is installed (oriented along the undeformed axis of the column) the greater control of critical load and transversal displacements can be obtained. Interesting results were obtained when the point of the force application (nut position) is close to the half of the total length of the system $\zeta \approx 0.5$. Close to the half of total length of the system when the spring stiffness is greater than zero $c > 0$ the presented curves on a plane λ_{cr} - ζ reach the local extreme - maximum. The maximum magnitude depends on spring stiffness. The maximum which is present at different stiffness of translational spring does not occur at the same value of the ζ parameter. When an increas of spring stiffness takes place the maximum point which corresponds to the critical force occurs at lower ζ.

4. Conclusions

In this study the system composed of a nut which moves along threaded rod was modeled. This system was modeled as a column subjected to Euler's load applied between the ends. The mathematical model takes into account the longitudinal elasticity of the support at one end. The study was carried out with numerical simulations with regard to the critical load of a column. On the basis of numerical simulations it was shown that the critical load of the considered system strongly depends on the stiffness of the applied longitudinal elastic support. An increase of the stiffness of the elastic element causes an increase of the loading capacity of the system. Between the ends of the column such a position of the point of force application (nut position) can be observed at which the extremum - maximum is present. The extreme point location depends on the spring stiffness which affects the longitudinal displacement of the column. During the design of threaded rod – moveable nut systems it must be taken into account that the change of critical load affects the critical rotational velocity which is not constant along the rod.

Acknowledgement

The study has been carried out within the statutory funds of the Czestochowa University of Technology (BS/PB-1-101-3020/11/P).

References

Kounadis, A.N. (1981) Divergence and flutter instability of elastically restrained structures under follower forces, Int. J. Engng. Sci. 19(4), 1981, 553-562.

Kounadis, A.N. (1983) The existence of regions of divergence instability for nonconservative systems under follower forces, Int. Journal Solids Structures 19(8), 1983, 725-733.

Ryu, J.B., Sugiyama, Y., Yim, K.B. and Lee, G.S. (2000) Dynamic stability of an elastically restrained column subjected to triangulary distributed subtangential forces, Computers & Structures 76, 2000, 611-619.

Sokół, K. (2014) Linear and Nonlinear Vibrations of a Column with an Internal Crack, Journal of Engineering Mechanics 140(5), http://dx.doi.org/10.1061/(ASCE)EM.1943-7889.

Sokół, K. and Uzny, S. (2016) Instability and vibration of multi-member columns subjected to Euler's load, Archive of Applied Mechanics, 86 (5),883–905, DOI: 10.1007/s00419-015-1068-6.

Sundararajan, C. (1976) Influence of an elastic end support on the vibration and stability of Beck's column, Int. J. Mech. Sci. 18, 1976, 239-241.

Tomski, L. (1985) Prebuckling Behaviour of Compound Column – Direct Nonlinear Analysis, Z. Angew. Math. U. Mech., 65(1), 59-61.

Tomski, L. and Kukla, S. (1989) Free Vibrations of a Certain Geometrically Nonlinear System with Initial Imperfection, AIAA Journal 28(7) 1240-1245.

Uzny, S., Sokół, K. and Osadnik, M. (2016) Free vibrations of the partially tensioned geometrically non-linear system subjected to Euler's load, Vibrations in Physical Systems 27, 399-406.

Uzny, S. (2011) Local and global instability and vibrations of a slender system consisting of two coaxial elements, Thin-Walled Structures, 49, 618-626.