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WEAK AXIS BUCKLING - ELASTIC RESISTANCE OF A COLUMN

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Abstract: *A solid finite element model for lateral buckling of column was developed in Ansys software. It was tested for calculation of elastic resistance. Series of random realizations for a long range of non-dimensional slenderness were created for this. As average relative differences as correlation between analytical results and results from Ansys software is presented.*

Keywords: Lateral buckling, Column, Imperfection, Elastic resistance, Steel.

1. Introduction

It is common to model lateral buckling and lateral-torsional buckling problems using shell elements instead of solid elements. However, there are some undesirable effects associated with shell elements, which may have an influence on the final resistance e.g. the material overlap at the web-flange junction (Jönsson and Stan, 2017). Another disadvantage of using shell elements can be a problematic modelling of varying thickness of a cross-section. Therefore, a solid model in Ansys software was developed.

2. Finite element model

The finite element research was performed on a model of a simply supported column subjected to a centric load on one end. European hot-rolled steel I200 cross-section was used for the column. Generally, the I200 is defined by seven dimensions h, b, t_1, t_2, R_1, R_2 and α, see Fig. 1a. However, its geometry was simplified to ensure the finite element mesh would be regular. Thus all fillets were removed. Their influence on load-carrying capacity was found neglectable (Kaim, 2004). The simplified cross-section is depicted in Fig. 1b.

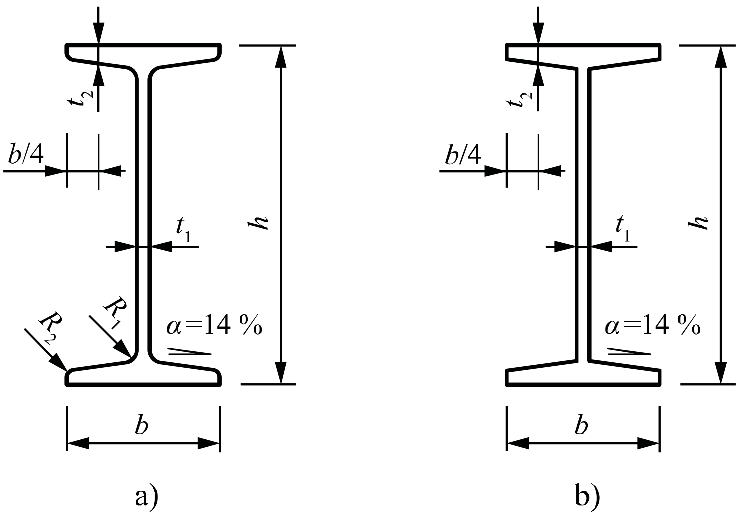

Fig. 1: Cross-section: a) real, b) idealized.

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The computational model was created in Ansys APDL software. Homogeneous structural solid element SOLID185 was used for the model. It is an 8-node solid element that is suitable for 3D modelling of solid structures having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The enhanced strain formulation was considered. This formulation prevents shear locking in bending-dominated problems and volumetric locking in nearly incompressible cases. The element introduces nine internal degrees of freedom to handle shear locking, and four internal degrees of freedom to handle volumetric locking. All internal degrees of freedom are introduced automatically at the element level and condensed out during the solution phase of the analysis (Ansys, 2014).

The meshing was performed similarly to the lateral-torsional buckling analysis (Kala and Valeš, 2017). The boundary conditions of the member end are generally taken from (Kala and Kala, 2011), however, a number of specific problems relating to elements SOLID185 is modelled more detailed and sophisticated.

2.1. Geometrical imperfection and elastic resistance

Initial deflection of the column was assumed to be a half sine wave with the amplitude e_0, as shown in Fig. 2.

$$v_0 = e_0 \sin\left(\frac{\pi x}{L}\right) \tag{1}$$

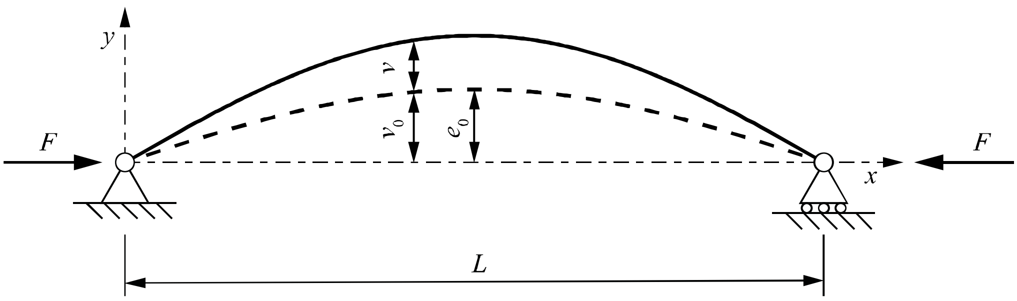

Fig. 2: Simply supported column with initial imperfection.

It is a common choice for the representation of imperfections (Boissonade et al., 2006). An additional deflection $v(x)$ appears on the column, when the axial force F is applied on it. Since the boundary conditions are considered as $x = 0$: $v + v_0 = 0$ and $x = L$: $v + v_0 = 0$, the additional deflection can be written as

$$v(x) = K \sin\left(\frac{\pi x}{L}\right) \tag{2}$$

where K is the maximum value of the additional deflection at mid-span. The classical elastic flexural equilibrium equation, accounting for the initial imperfection, becomes:

$$v'' + \frac{F\left(v_0 + v\right)}{EI_z} = 0 \tag{3}$$

where I_z is the second moment of area to axis z. Replacing Eq. (1) and Eq. (2) in Eq. (3) leads to the expression of K as follows:

$$K = \frac{F}{F_{cr} - F} e_0 \tag{4}$$

where N_{cr} is the critical flexural buckling load

$$N_{cr} = \frac{\pi^2 EI}{L^2} \tag{5}$$

The total column deflection at mid-span is

$$v_{\max} = \frac{e_0}{1 - \frac{F}{F_{cr}}} \tag{6}$$

The maximum stress $\sigma_{\max}$ due to compression and bending is

$$\sigma_{\max} = \frac{R}{A} + \frac{R\left|v_{\max}\right|}{W_z} = f_y \tag{7}$$

R represents the load-carrying capacity. It is the maximum load action F of elastic member, which is obtained when $\sigma_{\max}$ is equal to yield strength f_y. A is the area of the cross-section and W_z is the sectional module to axis z. Replacing Eq. (6) in Eq. (7) and isolating expression for R, we get the formula for elastic resistance published in (Kala, 2009) as

$$R = \frac{AD + F_{cr}W_z - \sqrt{A^2D^2 + 2AF_{cr}W_z\left(\left|e_0\right|F_{cr} - f_yW_z\right) + F_{cr}^2W_z^2}}{2W_z} \tag{8}$$

where D is a substitution

$$D = \left|e_0\right|F_{cr} + f_yW_z \tag{9}$$

3. Random input quantities

In general, the resistance R is a random quantity. It is a function of random geometrical and material characteristics (Melcher et al., 2004) and it can be studied using simulation methods, e.g. Latin Hypercube Sampling method (LHS) (McKey et al., 1979). A comparative analysis was performed for 13 selected non-dimensional slendernesses $\bar{\lambda}_z \in \left(0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1.0; 1.1; 1.2; 1.3; 1.4; 1.5; 1.6\right)$. 400 simulation runs were generated. Material characteristics of steel grade S235 and geometrical characteristics of the cross-section I200 based on (Melcher et al., 2004, Kala et al., 2009) were the random input quantities, see Tab. 1. All input random quantities were mutually statistically independent. The Gaussian probability distribution was considered for all of them. All other geometric and material characteristics were considered by their nominal values (h = 200 mm, b = 90 mm, t_1 = 7.5 mm, Poisson's ratio ν = 0.3). Residual stress was not considered in this analysis.

Tab. 1: Random quantities.

Quantity	Symbol	Mean value	Std. deviation
Flange thickness	t_2	11.3 mm	0.518 mm
Yield strength	f_y	297.3 MPa	16.8 MPa
Modulus of elasticity	E	210 GPa	10 GPa
Initial imperfection	e_0	0	1.5L/1960

4. Conclusions

A stochastic analysis of elastic resistance of compressed steel columns was carried out. The analysis was performed without the effect of residual stress. A linear stress-strain relationship of steel was used. Tab. 2 shows the correlation and average relative difference between elastic resistances from the analytical solution and the results calculated by Ansys.

Except of the slenderness $\bar{\lambda}_z$ = 1.6 all the average relative differences are positive which means that the analytical resistances are in average higher than those calculated by Ansys. Correlation between results is almost 1.0 for each slenderness. Such a good match confirms the accuracy of the finite element model.

This advanced solid model is suitable for subsequent analysis where residual stress and material nonlinearity will be considered. A specific instance where solid elements are more appropriate is the modelling of members in which the variation of residual stresses through plate thickness is non-negligible, while shell elements should be used for slenderer sections for which local imperfections affect the load carrying capacity.

Tab. 2: Correlation and average relative difference between elastic resistances.

Nondim. Slenderness $\bar{\lambda}_z$ [-]	**Correlation**	**Average relative difference [%]**
0.4	0.999 98	1.23
0.5	0.999 99	1.14
0.6	0.999 99	1.01
0.7	0.999 99	0.87
0.8	0.999 98	0.68
0.9	0.999 96	0.49
1.0	0.999 91	0.32
1.1	0.999 95	0.23
1.2	0.999 89	0.14
1.3	0.999 86	0.07
1.4	0.999 84	0.04
1.5	0.999 89	0.02
1.6	0.999 76	-0.03

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