23rd International Conference
ENGINEERING MECHANICS 2017
Svratka, Czech Republic, 15 – 18 May 2017

MATHEMATICAL MODEL OF ENGINE VALVE MECHANISM

T. Zvolský*

Abstract: *The paper deals with mathematical modeling and numerical computing of valve mechanism. There is described the computing method for a specific combustion engine and valve mechanism. Model computes with real mass and stiffness of valve mechanism parts. Mathematical model reflects the stiffness of the roller finger follower and deals with follower oscillation and consequently oscillation of the valve. Valve bounce from its seat, axial compliance of valve and forces from the flue-gas acting on valve is not included in this model. Valve lift, velocity and acceleration are graphically shown.*

Keywords: Valve, Engine, Model, Stiffness, Lagrange.

1. Introduction

Car combustion engine can typically operate in a wide speed range. At low speed, the engine has low fuel consumption and at high speed, the engine has high power. At high engine speed, it is necessary to quickly open and close the valves of the engine. There is an extreme accelerations and stress of the valve mechanism parts. At the same time there are requirements for high reliability and durability of the valve mechanism. Therefore it is necessary to optimize the valve mechanism. Optimization is based on the mathematical model, but it is not included in this paper.

2. Methods

This publication deals with the valve mechanism simulation of the combustion engine VW 1.6 MPI, series EA211. Valve train layout is DOHC - Double Over Head Camshaft, which is characterised by two camshafts located within the cylinder head. One controls the intake valves and the other one controls the exhaust valves. The camshaft moves the valve through a roller finger follower (Scheidt, 2014). Return movement of the valve into its seat is ensured by the spring. Valve clearance is eliminated by hydraulic lash adjuster. Mathematical model reflects the stiffness of the roller finger follower and deals with follower oscillation and consequently oscillations of the valve. DOHC valve train configuration with roller finger followers is shown in Fig. 1.

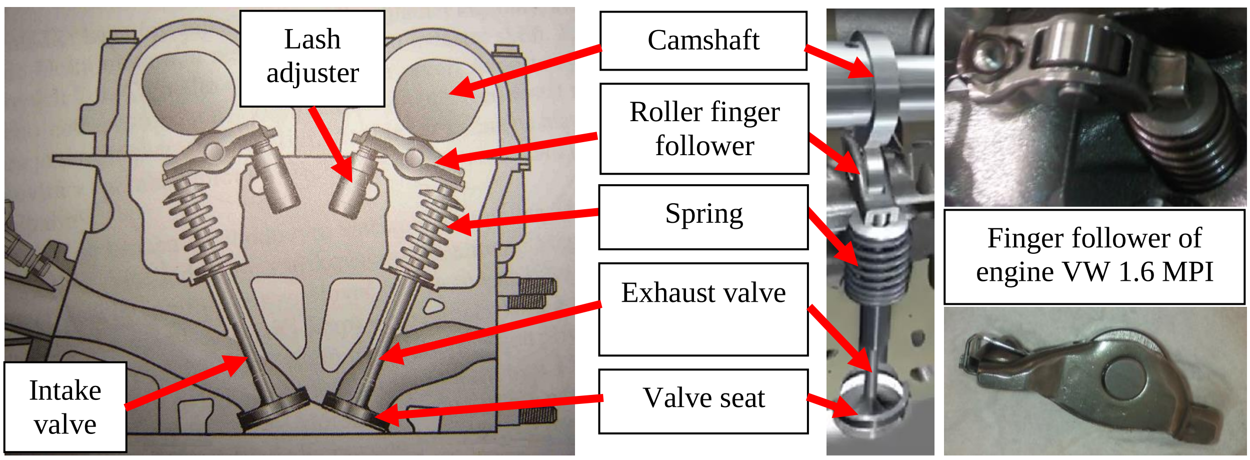

Fig. 1: Valve mechanism in combustion engine head.

* Ing. Tomáš Zvolský: Technical University of Liberec, Studentská 2; 461 17, Liberec; CZ, tomas.zvolsky@tul.cz

2.1. Mathematical model of valve mechanism

Mathematical model was created from real model of valve mechanism of gasoline VW engine 1.6 MPI, series EA211, shown in Fig. 2.

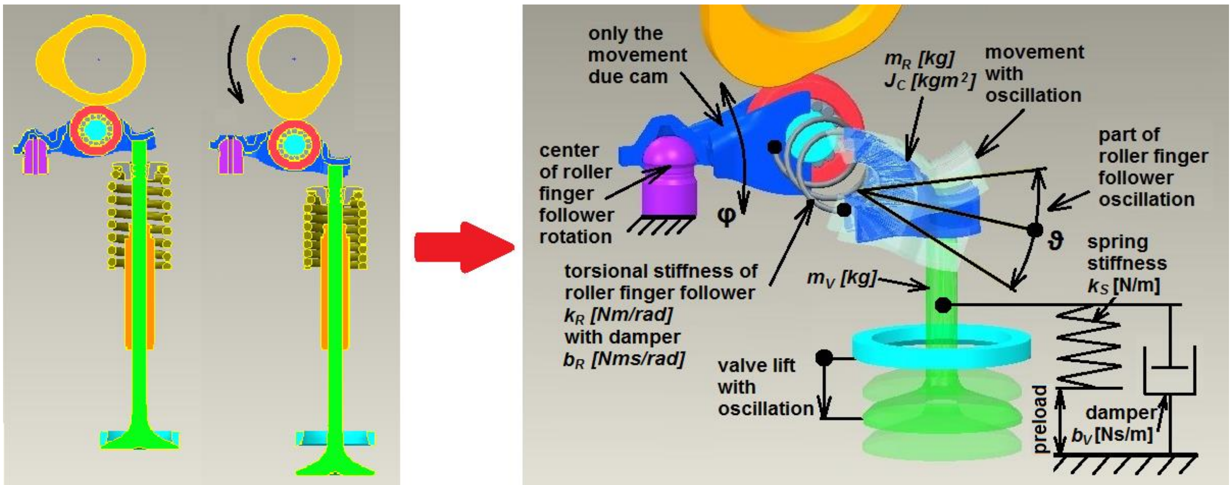

Fig. 2: Real and mathematical model of valve mechanism.

Roller finger follower is divided into two rigid parts that are elastically connected to each other. The first part of the finger follower is rotationally mounted to the spherical surface of the hydraulic lash adjuster and it includes a roller which is in contact with the cam. The second part of the finger follower is rotatable connected to the first part and can be deflected against it. Between both parts of the finger follower is inserted torsional stiffness k_R. The end of the finger follower is in contact with valve, which performs direct motion. Return movement of the valve is ensured by the spring. Stiffness of the spring is k_S. Masses m_V and m_R, moment of inertia J_C and stiffness of the finger follower k_R causes oscillation of the second part of the finger follower and consequently oscillations of the valve. The mathematical model is supplemented with natural damping b_R and b_V. Fig. 3 describes the dimensions of the valve mechanism and roller finger follower angular waveforms φ and ϑ. Angle φ is an input value in the mathematical model and angle ϑ is an output unknown variable.

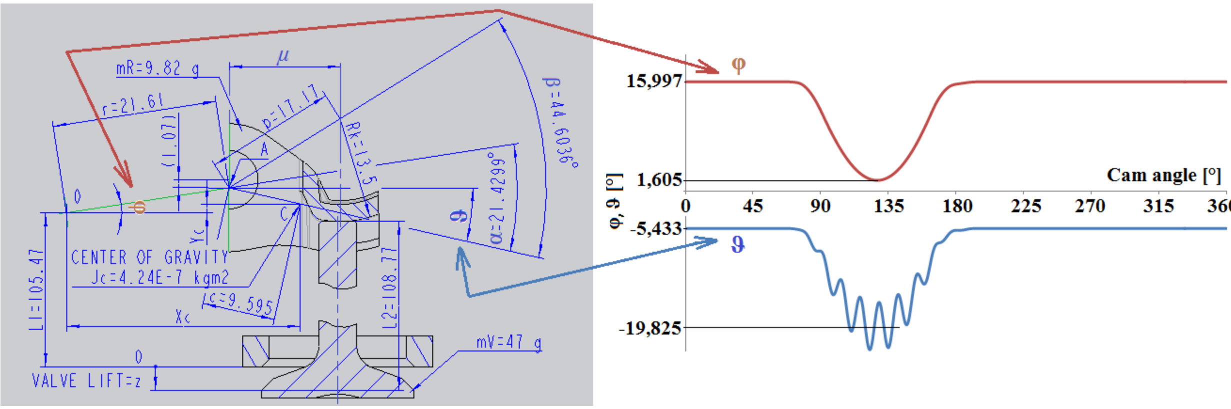

Fig. 3: Dimensions of valve mechanism with roller finger follower angular waveforms φ and ϑ.

Due to the cam rotation, the first part of the finger follower is deflected from the horizontal plane around point O by an angle φ. Since the second part of the finger follower is elastically connected with the first part, the second part of the finger follower is deflected from the horizontal plane by different angle ϑ. The second part of the finger follower can rotate around the point A. Point A also belongs to the first part of the finger follower and therefore trajectory of point A is a circle with center in point O and radius r. The second part of the finger follower has a center of gravity at point C, mass m_R and moment of inertia J_C. Valve mass, mass of fixing valve to the spring and 1/3 spring mass are included in m_V. Because this publication deals only with the valve oscillation, we do not need to specify the mass of the first part of the finger follower. It should be noted that permanent contact between cam and roller of finger follower is important assumption. Verification of this assumption is not described in this publication.

Stiffness of the finger follower was determined using the finite element method. The finger follower is considered as beam, which is constrained on two places - the first in the center of the spherical surface of hydraulic lash adjuster and the second in the center of roller. Load is placed to contact between finger follower and valve. Torsional stiffness of finger follower k_R can be calculated using bending stiffness k_O

$$k_R = k_O \xi^2 = \frac{F}{\delta} \xi^2 \tag{1}$$

where F is load, δ is displacement and ξ is distance between load position and center of roller.

2.2. Determining the equation of motion

Lagrange's equation of the second kind (Lanczos, 1986 and Julis, 1987) was used for solving this model. The angle $\varphi(t)$ is specified as a time waveform. It is an input value in the mathematical model and period depends on the speed of the camshaft. The output angle $\vartheta(t)$ is an unknown variable.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\vartheta}}\right) - \frac{\partial T}{\partial \vartheta} = -\frac{\partial V}{\partial \vartheta} - \frac{\partial D}{\partial \dot{\vartheta}} \tag{2}$$

The model has one degree of freedom. It is necessary to express total kinetic energy (T), potential energy (V) and Rayleigh dissipation function (D) of the system through the angle ϑ or its time derivatives. The kinetic energy of the second part of the finger follower consists of the translational and rotational parts.

$$\begin{aligned} T &= \frac{1}{2} m_R \left(\dot{x}_C^2 + \dot{y}_C^2\right) + \frac{1}{2} J_C \dot{\vartheta}^2 + \frac{1}{2} m_V \dot{z}^2 = \\ &= \frac{1}{2} m_R \left[r^2 \dot{\varphi}^2 + c^2 \dot{\vartheta}^2 + 2rc\dot{\varphi}\dot{\vartheta}\cos(\varphi - \vartheta)\right] + \frac{1}{2} J_C \dot{\vartheta}^2 + \frac{1}{2} m_V \left(r\dot{\varphi}\cos\varphi + \mu\dot{\vartheta}\right)^2 \end{aligned} \tag{3}$$

The kinetic energy of linear motion of the valve is expressed as a function of the angle ϑ using μ function.

$$\mu(\vartheta) = p\cos(\vartheta + \beta) \tag{4}$$

Potential energy of the system is determined by the elasticity of the finger follower and the spring.

$$V = \frac{1}{2} k_R (\vartheta - \varphi + \alpha)^2 + \frac{1}{2} k_S (z + preload)^2 \tag{5}$$

where z is valve lift. Opening the valve represents a positive value of the valve lift.

$$z = R_K - L_1 + L_2 - r\sin\varphi - p\sin(\vartheta + \beta) \tag{6}$$

Rayleigh dissipation function is given by equation (7).

$$D = \frac{1}{2} b_R \left(\dot{\vartheta} - \dot{\varphi}\right)^2 + \frac{1}{2} b_V \dot{z}^2 = \frac{1}{2} b_R \left(\dot{\vartheta} - \dot{\varphi}\right)^2 + \frac{1}{2} b_V \left(r\dot{\varphi}\cos\varphi + \mu\dot{\vartheta}\right)^2 \tag{7}$$

where b_R and b_V are damping coefficients. For natural damping we can use equations (8) and (9)

$$b_R = \frac{\ln 2}{\pi} \sqrt{k_R \left(J_C + m_R c^2\right)} = \frac{\ln 2}{\pi} \sqrt{k_R J_A} \tag{8}$$

$$b_V = \frac{\ln 2}{\pi} \sqrt{k_S m_V} \tag{9}$$

After expressing energy and calculating the relevant derivatives, we obtain differential equation.

$$\begin{aligned} \left(m_R c^2 + J_C + m_V \mu^2\right)\ddot{\vartheta} &= m_R rc\dot{\varphi}^2 \sin(\varphi - \vartheta) - m_R rc\ddot{\varphi}\cos(\varphi - \vartheta) - k_R (\vartheta - \varphi + \alpha) - b_R \left(\dot{\vartheta} - \dot{\varphi}\right) + \\ &+ m_V r\mu\dot{\varphi}^2 \sin\varphi - m_V r\mu\ddot{\varphi}\cos\varphi - m_V \mu\upsilon\dot{\vartheta}^2 + k_S \mu(z + preload) - b_V r\mu\dot{\varphi}\cos\varphi - b_V \mu^2 \dot{\vartheta} \end{aligned} \tag{10}$$

where $\upsilon(\vartheta)$ is given by equation (11).

`

$$\upsilon(\vartheta) = -p\sin(\vartheta + \beta) \tag{11}$$

2.3. Solving the equation of motion

For solving the equation of motion was used Matlab software. The graphs below show lift, velocity and acceleration of the exhaust valve at camshaft speed 3250 rpm. This corresponds to the crankshaft speed 6500 rpm, which is the maximum motor speed. One complete rotation of the camshaft takes approximately 18.5 ms, but the graphs show only useful 7 ms. In those graphs are shown theoretical cam and real valve waveforms, specified in (Heisler, 1995).

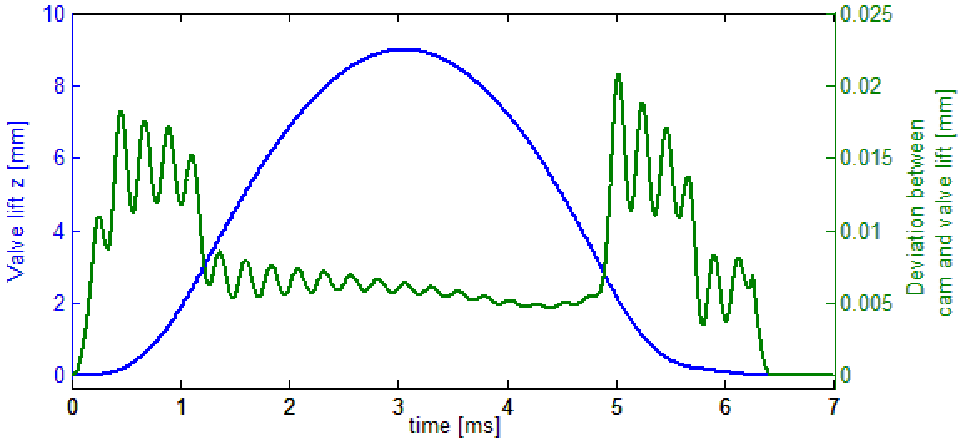

Fig. 4: Valve lift (blue) and deviation between theoretical cam lift and real valve lift (green).

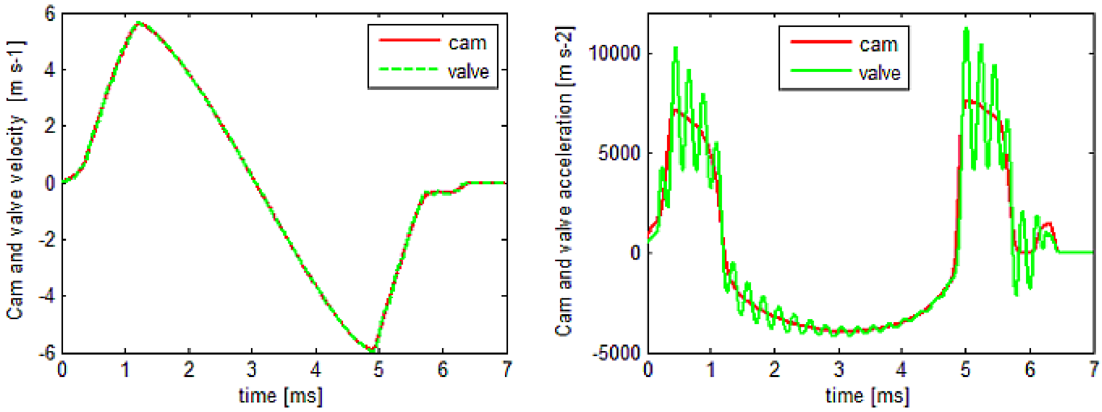

Fig. 5: Theoretical cam and real valve velocity and acceleration.

3. Conclusions

This publication deals with the valve mechanism simulation of the combustion engine 1.6 MPI, series EA211. Mathematical model reflects the stiffness of the roller finger follower and deals with follower oscillations and consequently oscillations of the valve. The stiffness of the finger follower is high and therefore frequency is relatively high, according to the simulation approximately 4000 Hz. Deviation between theoretical cam lift and real valve lift is greater than zero. This is important to keeping permanent contact between the valve and the finger follower. Theoretical cam velocity and real valve velocity are almost identical. Valve acceleration amplitude of oscillation is very significant.

Acknowledgement

This publication was written at the Technical University of Liberec with the support of the Specific University Research Grant, as provided by the Ministry of Education, Youth and Sports of the Czech Republic in the year 2017.

References

Heisler, H. (1995) Advanced Engine Technology. SAE Technology, pp. 2-40.

Julis, K. and Brepta, R. (1987) Mechanics part 2. Dynamics. SNTL, Praha (in Czech).

Lanczos, C. (1986) The Variational Principles of Mechanics. Dover Publications Inc., New York, pp. 111-119.

Scheidt, M. and Lang, M. (2014) Pure Efficiency. 10^{th} Schaeffler Symposium, pp. 43-55.