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PLANE CONTACT OF TWO ELASTIC SOLIDS WITH FUNCTIONALLY GRADED COATINGS JOINED BY AN IMPERFECT INTERFACE

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Abstract: *Plane contact problem on normal interaction of two dissimilar elastic solids is considered. The solids consist of a homogeneous semi-infinite substrate and a functionally graded coating joined by an imperfect interface to the substrate. The problem is reduced to the solution of a dual integral equation which is solved by the bilateral asymptotic method. Numerical results illustrating the difference between the ideal and imperfect interface of the coating and the substrate are provided.*

Keywords: contact, elasticity, functionally graded coating, imperfect interface, analytical method

1. Introduction

Most of the results in the field of contact mechanics of elastic solids with functionally graded (FG) and homogeneous coatings are obtained in assumption of complete adhesion (perfect coupling) at the interface between the coating and substrate (Guler and Erdogan, 2004, Ke and Wang, 2006). However, the coating–substrate adhesion depends on the technique of coating application and on the existence of microdefects on this interface. Some results in contact interaction of solids with homogeneous and piecewise homogeneous coatings taking into account imperfect coating-substrate interface one can find in the papers by Antonenko and Velichko (2014); Lipton (2001); Torskaya and Goryacheva (2003) and We et al. (2013), etc. The present paper addresses to the study of contact interaction of coated FG solids.

2. Statement of the problem

Let us consider plane contact of two massive elastic parabolic solids with coatings, initially contacting at a point (0,0) of a Cartesian coordinate system (*x*,*z*). The solids are subjected to the normal centrally applied force *P* and move distances $-\delta_1$ and δ_2 along the *z*-axis. Hereafter indexes 1 and 2 correspond to the lower and upper solid, respectively. Displacements of the solid's surface can be expressed by the following equation:

$$z = 0 : w_1 - x^2 (2R_1)^{-1} + \delta_1 = w_2 + x^2 (2R_2)^{-1} - \delta_2 \tag{1}$$

where R_1 and R_2 are radii of curvature of the solids, *u* and *w* are displacements with respect to *x* and *z*.

Each solid consist of a homogeneous half-plane (substrate) with constant values of elastic moduli $E_i^{(s)}, \nu_i^{(s)}$ and a functionally graded coating of thickness H_i. Young's modulus and Poisson's ratio of the coating vary with depth according to the continuously differentiable functions $E_i^{(c)}(z), \nu_i^{(c)}(z)$. Hereafter,

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superscripts (*c*) and (*s*) correspond to the coating and to the substrate, respectively. Let us consider imperfect coupling of the coating and substrate:

$$z=(-1)^i H_i:\quad \sigma_z^{(c)}=\sigma_z^{(s)},\ w^{(c)}=w^{(s)},\ \tau_{zx}^{(c)}=\tau_{zx}^{(s)},\tau_{zx}^{(c)}=\frac{u^{(c)}-u^{(s)}}{e_i},\ i=1,2 \tag{2}$$

where σ_z and τ_{zx} are the components of the stress tensor, coefficients $e_i\in[0,\infty]$ characterizes the adhesion at the interface. The case of $e_i=0$ correspond to the comlete perfect coupling, i.e. $u^{(c)}=u^{(s)}$. The case of $e_i=\infty$ correspond to the full slip. Methods for determining this coefficient are discussed in (Goryacheva, 1998).

Outside of the punch, the surface is traction-free. It is required to determine the contact normal stresses under the punch: $\sigma_z^{(c)}\big|_{z=0}=p(x), x\le a$ and the radius of the contact area *a*.

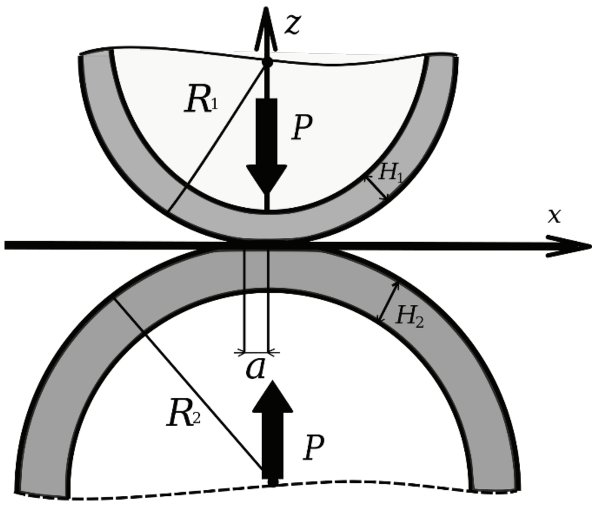

Fig. 1: Scheme of the contact problem

3. Solution of the problem

Using the Fourier integral transformation technique, similar to the axisymmetric formulation (Kudish et al., 2017), the dual integral equation of the problem is obtained:

$$\int_0^\infty L(\lambda\gamma)\frac{\bar{p}_0(\gamma)}{\gamma}\cos(\gamma x')d\gamma=\frac{\pi\Theta_c}{2a}\left(\delta_1+\delta_2-\frac{a^2x'^2}{2R}\right),\ |x'|\le 1 \tag{3}$$

$$L(\lambda\gamma)=\Theta_c\left(\frac{L_1(\lambda\gamma)}{E_1'^{(c)}}+\frac{L_2(\lambda\mu\gamma)}{E_2'^{(c)}}\right),\quad \Theta_c=\left(\frac{1}{E_1'^{(c)}}+\frac{1}{E_2'^{(c)}}\right)^{-1},\ R=\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1} \tag{4}$$

$$E_i'^{(c)}=E_i^{(c)}(0)/\left(1-\nu_i^{(c)}(0)\right),\quad E_i'^{(s)}=E_i^{(s)}/\left(1-\nu_i^{(s)}\right) \tag{5}$$

where $x'=x/a$ is the dimensionless coordinate; $\lambda=H_1/a$ is the relative thickness of the lower coating; $\mu=H_2/H_1$ is the ratio of the coating's thicknesses; $E_i'^{(c)}$ are the effective elastic moduli on the surface of the coatings; $\bar{p}_0(\gamma)$ is the Fourier transform of the dimensionless contact stresses $p_0(x')=p(ax')$; $L_i(\lambda\gamma)$ are the compliance functions of the lower and upper solids. Compliance functions are defined as solutions of two-point boundary value problems for a system of ordinary differential equation with variable coefficients, which can be obtained similar to (Vasiliev et al., 2017a).

To illustrate how the imperfect coupling of the coating and substrate influence the fundamental characteristics of the contact let us consider following example:

$$\nu_i^{(c)}(z) \equiv \nu_i^{(s)} = 0.3, \quad E_1^{(s)} = E_2^{(s)} = E_0, \quad \mu = H_2/H_1 = 0.25, \quad e_1 = e_2$$
$$E_1^{(c)}(z) = 0.5E_0(1 - z/H_1), -H_1 \le z \le 0; \quad E_2^{(c)}(z) = E_0(2 - z/H_2), 0 \le z \le H_2 \tag{6}$$

Figure 2 contain the graphs of the kernel transform $L(\lambda\gamma)$ for different values of parameters e_i. It is seen that the adhesion coefficient sufficiently change the behavior of the kernel transform. As it was shown earlier behavior of the kernel transform significantly affects the behavior of the most important characteristics of the contact such as contact stresses, size of the contact area, contact stiffness, etc., see (Vasiliev et al., 2017b) for details.

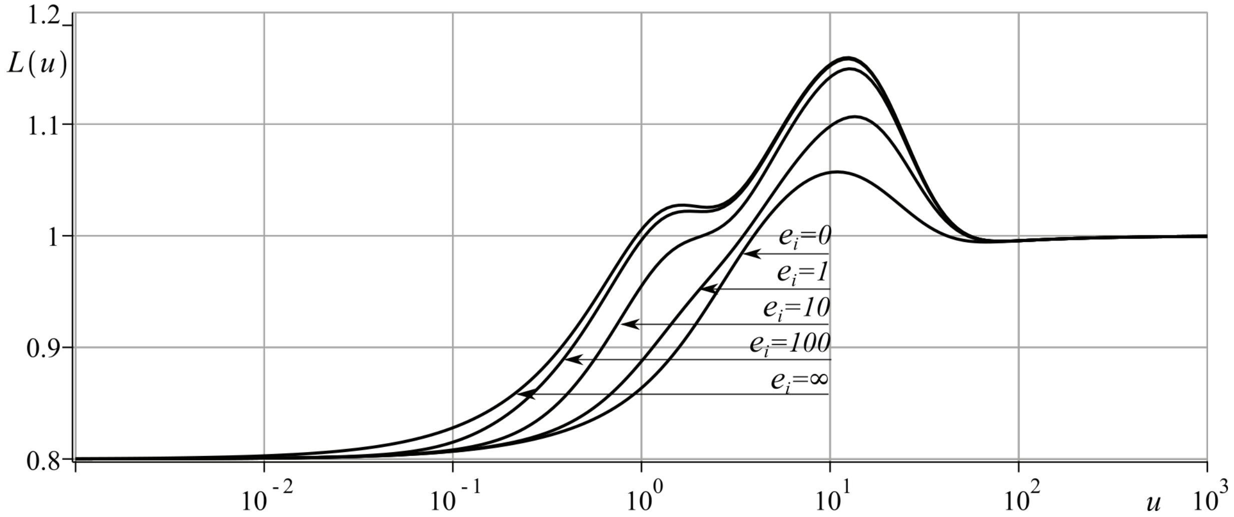

Fig. 1: Kernel transform for different values of adhesion coefficient e_i=0, 1, 10, 100 and ∞.

Solution of the dual integral equation (3) is constructed using the bilateral asymptotic method. Kernel transform $L(\lambda\gamma)$ is approximated by the following expression for a fixed value of μ:

$$L(\lambda\gamma) \approx \Pi_N(\lambda\gamma) = \prod_{i=1}^{N} \left(\lambda^2\gamma^2 + A_i^2\right) / \left(\lambda^2\gamma^2 + B_i^2\right), \quad A_i, B_i \in C. \tag{7}$$

Earlier it was shown for the vast majority of types of depth-wise variation of elastic moduli that an approximation with relative error less that 0.5% can be constructed.

Taking into account classical assumption for the contact stresses at the boundary of the contact area: $p_0(1) = 0$, similar to (Vasiliev et al., 2017) approximated analytical expressions for the contact stresses are obtained:

$$p_0(x') = \frac{2P}{a\pi}\left[\sqrt{1 - x'^2} + \sum_{i=1}^{N} C_i \left(2\frac{\lambda}{A_i}\sqrt{1 - x'^2} - Z\left(\frac{A_i}{\lambda}, x'\right)\right)\right] \tag{8}$$

$$Z(a, x) = \int_x^1 \frac{t\cosh(a(t - x)) I_0(a)/I_1(a) - \sinh(a(t - x))}{\sqrt{1 - t^2}} dt \tag{9}$$

Parameters C_i are the solution of the following system of linear algebraic equations:

$$\sum_{i=1}^{N} C_i \left(\frac{B_k I_0(A_i\lambda^{-1})/I_1(A_i\lambda^{-1}) + A_i K_0(A_i\lambda^{-1})/K_1(A_i\lambda^{-1})}{A_i^2 - B_k^2} + 2\lambda\right) = -\frac{1}{B_k}, \; k = 1..N \tag{10}$$

Half width of the contact area satisfy following equation:

$$a^2 = \frac{4PR}{\pi\Theta_s}\left(1 + 2\lambda\sum_{i=1}^{N}\frac{C_i}{A_i}\right) \quad (11)$$

Expressions (8) and (11) are asymptotically exact for small and large values of dimensionless coating thickness λ.

4. Conclusions

Two-dimensional (plane) contact problem on interaction of two dissimilar elastic solids with FG coatings is considered under the assumption, that coating–substrate interface is imperfect. Approximated analytical expressions for the contact stresses and size of the contact area are obtained. Using these results internal stresses within the coating and substrate can be easily calculated, see (Volkov et al., 2016) for details. The results can be generalized to the case of elastohydrodynamic lubricated contact (Kudish et al., 2016), electroelastic piezoelectric materials (Volkov et al., 2017) or to the case of bending of a flexible element (beam, plate) lying of a FG coating-substrate system (Volkov and Vasiliev, 2014).

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