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COMPARISON OF PHONONIC STRUCTURES WITH PIEZOELECTRIC $0.62PB(MG_{1/3}NB_{1/3})O_3$-$0.38PBTIO_3$ DEFECT LAYERS

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Abstract: *Three types of multilayer structures were analyzed in the work: binary (periodic), Thue-Morse (quasiperiodic) and Severin (aperiodic). Layers in the structures were made of water and epoxy. In the given structure occurring layer were changed sequentially to layer made of a 0.2mol% Fe-doped relaxor-based ferroelectric $0.62PB(MG_{1/3}NB_{1/3})O_3$-$0.38PBTIO_3$ (PNM-0.38PT). The tests were carried out for the ultrasonic frequency range from 500 kHz to 700 kHz. As part of the calculations, the transmission characteristics of structures with defects were obtained. In Thue-Morse structure all the characteristics were different, but in the binary and Severin structures, the same transmission values were observed for different types of structures.*

Keywords: phononic, defect, transfer marix, aperiodic, multilayers

1. Introduction

Composite materials such as quasi one-dimensional multilayer structures exhibit very interesting phononical properties related to the lack of propagation of acoustic waves in a given frequency range depending on their structure, and therefore are used in a wide range of acoustic devices (Badreddine et al. (2014), Han et al. (2012)). One of the methods allowing to determine the properties of these structures is the Transfer Matrix Method (TMM) algorithm (Garus and Sochacki (2017)), which was used in this work. Other algorithms are, for example, PWE (Kushwaha et al. (1994), Vasseur et al. (2012)) and FDTD (Miyashita (2005), Alagoz et al. (2009), Sigalas and Economou (1992)).

2. Methods

All calculations were performed using the Transfer Matrix Method (TMM). Acoustic wave propagation can be described by

$$\frac{1}{v_i^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0 \tag{1}$$

where p is the pressure in the analyzed medium, t is a time and v_i is phase velocity in i layer. The solution of Eq. (1) for f frequency in the one dimensional case can be calculated as

$$p_i\left(x\right) = A_i e^{i\frac{2\pi f}{v_i}x} + B_i e^{-i\frac{2\pi f}{v_i}x} \tag{2}$$

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where forward wave represents A_i coefficient and B_i the reflected wave. For d_i thickness and ρ_i mass density of layer i the transmission from Transfer Matrix Method can be described as

$$T = \left| \left[M_{in,1} \left(\prod_{i=1}^{n-1} \begin{bmatrix} e^{i\frac{2\pi f d_i}{v_i}} & 0 \\ 0 & e^{-i\frac{2\pi f d_i}{v_i}} \end{bmatrix} M_{i,i+1} \right) \begin{bmatrix} e^{i\frac{2\pi f d_n}{v_n}} & 0 \\ 0 & e^{-i\frac{2\pi f d_n}{v_n}} \end{bmatrix} M_{n,out} \right]_{1,1} \right|^{-2} \tag{3}$$

The $M_{i,j}$ is transfer matrix between layers i and j which is defined by

$$M_{i,j} = \begin{bmatrix} \dfrac{\rho_i v_i + \rho_j v_j}{2\rho_i v_i} & \dfrac{\rho_i v_i - \rho_j v_j}{2\rho_i v_i} \\ \dfrac{\rho_i v_i + \rho_j v_j}{2\rho_i v_i} & \dfrac{\rho_i v_i + \rho_j v_j}{2\rho_i v_i} \end{bmatrix} \tag{4}$$

3. Research

Tab. 1: Material properties (Wang et al. (2014)).

Symbol	Name	Mass density [kg/m^3]	Phase velocity [m/s]	Layer thickness [mm]
A	Water	1000	1480	5
B	Epoxy	1180	2535	5
F	PNM-0.38PT	8093	4410	5

Three types of structures were analyzed in the work: $\boldsymbol{X_L^B}$ - binary (periodic), $\boldsymbol{X_L^{T-M}}$ - Thue-Morse (quasiperiodic), $\boldsymbol{X_L^S}$ - Severin (aperiodic). They were chosen so that the total length of each structure was equal (they all consisted of eight layers of the same thickness). The material parameters used for calculations were collected in Table 1. In each case the medium surrounding the structure was water. The research was carried out for the range of ultrasonic waves from 500 kHz to 700 kHz. The layer made of PNM-0.38PT replaced subsequent layers in the analyzed structures. The examined structures were collected in Table 2. The frequency *f [kHz]* and transmission percentage *T [%]* of the peaks have been determined and shown in Table 3.

Tab. 2: Defect layer distributions in the analyzed structures (binary - X_L^B, Thue-Morse - X_L^B, Severin - X_L^B), L is number of defect layer.

$\boldsymbol{L}$	$\boldsymbol{X_L^B}$	$\boldsymbol{X_L^{T-M}}$	$\boldsymbol{X_L^S}$
1	FBABABAB	FBBABAAB	FBABBBAB
2	AFABABAB	AFBABAAB	AFABBBAB
3	ABFBABAB	ABFABAAB	ABFBBBAB
4	ABAFABAB	ABBFBAAB	ABAFBBAB
5	ABABFBAB	ABBAFAAB	ABABFBAB
6	ABABAFAB	ABBABFAB	ABABBFAB
7	ABABABFB	ABBABAFB	ABABBBFB
8	ABABABAF	ABBABAAF	ABABBBAF

Tab. 3: Frequencies and the transmission values where the studied structures reach their maximum.

Structure		Peak number					
		1	2	3	4	5	6
X_1^B	$f\,[kHz]$	534.75	577.85	600.86	624.35	653.19	694.02
	$T\,[\%]$	2.15	21.16	3.3	2.81	70.02	1.45
X_5^B	$f\,[kHz]$	503.97	585.4	651.18			
	$T\,[\%]$	3.36	29.86	74.48			
X_2^B, X_8^B	$f\,[kHz]$	535.98	587.22	609.44	631.07	691.36	
	$T\,[\%]$	2.25	5.07	2.97	3.12	1.5	
X_4^B, X_6^B	$f\,[kHz]$	503.11	593.73	624.59			
	$T\,[\%]$	3.46	5.65	3.1			
X_3^B, X_7^B	$f\,[kHz]$	523.48	579.78	613.7	652.9		
	$T\,[\%]$	2.5	19.06	8.89	69.87		
X_1^{T-M}	$f\,[kHz]$	545.67	566.69	600.77	621.72	655.41	687.93
	$T\,[\%]$	3	20.96	3.31	6.91	1.31	18.05
X_2^{T-M}	$f\,[kHz]$	510.26	546.57	582.41	609.18	648.14	670.45
	$T\,[\%]$	3.34	1.71	14.87	3.17	25.14	2.75
X_3^{T-M}	$f\,[kHz]$	510.83	548.04	594.36	618.9	666.75	
	$T\,[\%]$	3.35	1.66	10.07	16.13	1.88	
X_4^{T-M}	$f\,[kHz]$	537.99	589.85	636.34	683.39		
	$T\,[\%]$	4.17	15.07	10.09	12.7		
X_5^{T-M}	$f\,[kHz]$	502.02	602.57	669.52			
	$T\,[\%]$	3.47	4.12	0.94			
X_6^{T-M}	$f\,[kHz]$	509.08	575.94	609.91	662.64		
	$T\,[\%]$	3.39	3.67	3.79	1.44		
X_7^{T-M}	$f\,[kHz]$	501.17	526.47	585.37	614.69		
	$T\,[\%]$	4.02	3.6	5.42	10.83		
X_8^{T-M}	$f\,[kHz]$	543.48	588.65	609.92	644.64	671.35	
	$T\,[\%]$	7.85	1.95	2.83	0.24	7.29	
X_1^S	$f\,[kHz]$	506.42	563.59	590.85	618.21	649.33	679.57
	$T\,[\%]$	3.34	9.29	6.05	3.39	26.85	5.61
X_5^S	$f\,[kHz]$	503.97	585.4	651.18			
	$T\,[\%]$	3.36	29.86	74.48			
X_2^S, X_8^S	$f\,[kHz]$	506.76	569.16	600.78	627.14	673.89	
	$T\,[\%]$	3.34	4.36	3.28	2.94	2.36	
X_4^S, X_6^S	$f\,[kHz]$	564.49	613.48	686.51			
	$T\,[\%]$	4.66	8.68	3.06			
X_3^S, X_7^S	$f\,[kHz]$	550.66	601.29	642.97			
	$T\,[\%]$	12.91	8.15	38.03			

4. Conclusions

Phononic structures are intensively studied in many centers around the world using many algorithms. At work using the TMM algorithm, the transmission structure of three types of multilayers was determined (binary, Thue-Morse and Severin). As a defect, a 0.2mol% Fe-doped relaxor-based ferroelectric $0.62PB(MG_{1/3}NB_{1/3})O_3$-$0.38PBTIO_3$ (PNM-0.38PT) layer was introduced into the structures. The use of this material allows to control, to a certain extent, the location of the transmission peaks as shown by (Wang *et al.* (2014)). In the analyzed range of ultrasonic wave frequencies, there were from three to six transmission peaks for all types of structures. Of the 78 specific peaks, only less than one third exceeded the 10% transmission threshold and only 4 were in the range of about 70%. For the Thue-Morse structure, the introduction of the defect resulted in different transmission characteristics for all L parameters. However, the most interesting results occurred for the binary and Severin structures where the introduction of the defect resulted in the same transmission characteristics for the pair of structures respectively $X_2^B = X_8^B$, $X_4^B = X_6^B$, $X_3^B = X_7^B$, $X_2^S = X_8^S$, $X_4^S = X_6^S$ and $X_3^S = X_7^S$. These structures for a small number of layers are closely related to each other. It would be worth conducting additional research for the greater complexity of the multilayers, where the similarity of the binary network is no longer present for the next generation of the Severin network. However, the fact that identical transmission structures exist for different types of defects in multilayers is interesting and worth analyzing more closely.

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