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APPLICATION OF MODAL SYNTHESIS TO DYNAMIC PARAMETERS MODIFICATION

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Abstract: *Undesirable vibration level of construction parts is a dangerous and unwanted phenomenon in the modern engineering which can potentially lead to damage or destruction of a construction component. In some cases, this issue can appear in already constructed constructions where additional reducing of the vibration level is a required operation. This reducing can be performed by dynamic parameters modification to avoid the resonance states or to increase the ratio of damping of competent structure modes. This work is aimed to presentation of modal synthesis application to desired modification of dynamic parameters of a structure to decrease the level of its vibration. It includes the theory of modal synthesis in case of original and modifying structures and the demonstration example of the the cantilever beam modified by aluminum foam structures in appropriate places to reach the desired modal and spectral parameters.*

Keywords: modal synthesis, structural dynamic modification, dynamic parameters, vibration level reducing, resonance

1. Introduction

A common occurrence in practical engineering are undesirable levels of vibration in the machinery structures which can influent their safety and reliability. The dynamic properties of the machines structure itself, to an extent, affects the level of vibration of each of its individual parts. In the research stage, it is now necessary to extensively analyse/synthesize the dynamic properties of the machine and its structure followed by the optimization of significant parameters.

A suitable concept must be chosen in the design of a structure, which must vibrate within acceptable levels during operation. For example, based on numeric analysis and optimization of individual structural components, it is possible to create a real structure which satisfies the operational conditions set upon it. But in general, the real structure partially exhibits differing properties than those predicted computationally, either due to inappropriate simplification or inaccurate physical or geometrical parameters. It is therefore necessary to modify critical structural elements to fulfil the desired properties.

This operation can be performed through the modal synthesis, where an original structure is modified by an additional component. This approach combines the modal properties of the real structure obtained through measurements and the modal properties of additional components obtained computationally or through measurements. Through optimization of the additional components it is possible to obtain the desired properties of a modified structure while reducing the computational requirements and increasing accuracy of the results.

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2. Modal synthesis method

This chapter is aimed to the modal synthesis explanation of the original structure and a substructure to determine modal parameters of the resulting system. The original structure is represented by the proportionally damped system whose simplified eigenvalue solution has the following form.

$$(\boldsymbol{K}_0 - \boldsymbol{M}_0\omega_{0j}^2)\boldsymbol{v}_{0j} = \boldsymbol{0} \tag{1}$$

Where stiffness and mass parameters are represented by coefficient matrices $\boldsymbol{K}_0$ and $\boldsymbol{M}_0$ and jth eigen mode and natural frequency for this mode by symbols $\boldsymbol{v}_{0j}$ and ω_{0j}.

Following equations describe conditions for orthonormality.

$$\boldsymbol{V}_0^T\boldsymbol{K}_0\boldsymbol{V}_0 = \boldsymbol{\Omega}_0^2,\ \boldsymbol{V}_0^T\boldsymbol{M}_0\boldsymbol{V}_0 = \boldsymbol{I},\ \boldsymbol{V}_0^T\boldsymbol{B}_0\boldsymbol{V}_0 = 2\boldsymbol{\Delta} = 2(\alpha\boldsymbol{I} + \beta\boldsymbol{\Omega}_0^2) \tag{2}$$

Where $\boldsymbol{\Omega}_0$ is the diagonal spectral matrix that includes natural frequencies ω_{0j}, $\boldsymbol{V}_0$ is the modal matrix including eigen modes $\boldsymbol{v}_{0j}$, $\boldsymbol{B}_0$ is the damping matrix and $\boldsymbol{\Delta}$ represents the matrix of constant decay elements δ_j, which are expressed by the following equation

$$\delta_j = 2\xi\omega_{0j} \tag{3}$$

The symbol ξ introduce the comparative damping of the structure.

Because modifying substructures are not located evenly along the original structure, the resulting structure can be interpreted as a disproportionally damped system of 2n dimensional space represented by the coefficient matrices N and P. The vibration of this system can be described by the following second order differential equation.

$$\boldsymbol{N}\dot{\boldsymbol{x}} - \boldsymbol{P}\boldsymbol{x} = \boldsymbol{r} \tag{4}$$

where

$$\boldsymbol{P} = \begin{bmatrix} -\boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M} \end{bmatrix},\ \boldsymbol{N} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{M} \\ \boldsymbol{M} & \boldsymbol{0} \end{bmatrix},\ \boldsymbol{x} = \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix},\ \boldsymbol{r} = \begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{0} \end{bmatrix} \tag{5}$$

Where coefficient matrices are represented by $\boldsymbol{K}, \boldsymbol{M}$ and $\boldsymbol{B}$ and the variables $\boldsymbol{q}, \boldsymbol{f}$ express generalised displacement vector and the excitation force.

Eigenvalue problem solution for this system:

$$(\boldsymbol{P} - s_j\boldsymbol{N})\boldsymbol{w}_j = \boldsymbol{0},\ (\boldsymbol{K} + s_j\boldsymbol{B} + s_j^2\boldsymbol{M})\boldsymbol{v}_j = \boldsymbol{0} \tag{7}$$

Where $\boldsymbol{v}_j$ interpret jth eigen mode and imaginary part of its eigenvalue s_j express the angular frequency of damped system ω_{Dj}.

The following equations represents conditions for orthonormality.

$$\boldsymbol{W^T P\, W = S} \qquad \boldsymbol{W^T N\, W = I}$$
$$\boldsymbol{S\, V^T M\, V\, S - V^T K\, V = S} \qquad \boldsymbol{V^T B\, V + V^T M\, V\, S + S\, V^T M\, V = I} \tag{8}$$

Where $\boldsymbol{W}$ and $\boldsymbol{V}$ are modal matrices and $\boldsymbol{S}$ represents the spectral matrix. These matrices can be expressed by relations:

$$\boldsymbol{W} = \{\boldsymbol{w}_j\} = \begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{V\,S} \end{bmatrix},\ \boldsymbol{V} = \{\boldsymbol{v}_j\},\ \boldsymbol{S} = diag(s_j) \tag{9}$$

Relations (8) and (9) can be used to determine modal-spectral parameters of the resulting structure using original structure modal matrices $\boldsymbol{V}_0$, $\boldsymbol{\Omega}_O$, $2\boldsymbol{\Delta}$ and coefficient matrices of the modifying structure $\boldsymbol{M}_N$, $\boldsymbol{B}_N$, $\boldsymbol{K}_N$. Because dimensions of original structure matrices are usually different then coefficient matrices of the modifying structure, reduction methods as Guyan method or SEREP and zero elements expansions are required (Avitable, 2002 and Curnier 1983). This process represents the modification of added substructure matrices $\boldsymbol{M}_N$, $\boldsymbol{B}_N$, $\boldsymbol{K}_N$ through $\boldsymbol{M}_R$, $\boldsymbol{B}_R$, $\boldsymbol{K}_R$ to $\boldsymbol{M}_A$, $\boldsymbol{B}_A$, $\boldsymbol{K}_A$, which can be graphically described as:

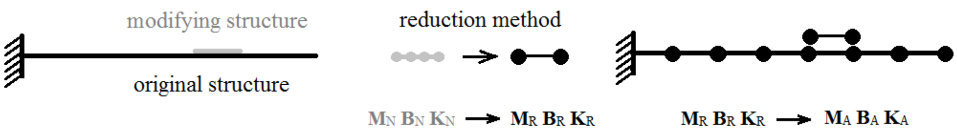


Fig. 1: Principal of modal synthesis of an original and modifying structure.

and express by equations:

$$\boldsymbol{M_A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{M_R} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \boldsymbol{K_A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{K_R} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \boldsymbol{B_A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{B_R} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

orthogonality conditions for disproportionally damped system:

$$[\boldsymbol{V^T}\ \boldsymbol{S}\ \boldsymbol{V}_L^T]\begin{bmatrix} -(\boldsymbol{K}_0 + \boldsymbol{K}_A) & 0 \\ 0 & \boldsymbol{M}_0 + \boldsymbol{M}_A \end{bmatrix}\begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{V}\ \boldsymbol{S} \end{bmatrix} = \boldsymbol{S}$$

$$[\boldsymbol{V^T}\ \boldsymbol{S}\ \boldsymbol{V}_L^T]\begin{bmatrix} -(\boldsymbol{K}_0 + \boldsymbol{K}_A) & 0 \\ 0 & \boldsymbol{M}_0 + \boldsymbol{M}_A \end{bmatrix}\begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{V}\ \boldsymbol{S} \end{bmatrix} = \boldsymbol{S} \tag{11}$$

substitution using transformation matrix $\boldsymbol{T}_L$:

$$\boldsymbol{V}_L = \boldsymbol{V}_O \boldsymbol{T}_L \tag{12}$$

condition for orthogonality then can be formed as:

$$[\boldsymbol{T}_L^T\ \boldsymbol{S}\ \boldsymbol{T}_L^T]\begin{bmatrix} -(\boldsymbol{\Omega}_0^2 + \boldsymbol{V}_0^T\boldsymbol{K}_A\boldsymbol{V}_0) & 0 \\ 0 & \boldsymbol{I} + \boldsymbol{V}_0^T\boldsymbol{M}_A\boldsymbol{V}_0 \end{bmatrix}\begin{bmatrix} \boldsymbol{T_L} \\ \boldsymbol{T_L}\ \boldsymbol{S} \end{bmatrix} = \boldsymbol{S}$$

$$[\boldsymbol{T}_L^T\ \boldsymbol{S}\ \boldsymbol{T}_L^T]\begin{bmatrix} 2\Delta_P + \boldsymbol{V}_0^T\boldsymbol{B}_A\boldsymbol{V}_0 & \boldsymbol{I} + \boldsymbol{V}_0^T\boldsymbol{M}_A\boldsymbol{V}_0 \\ \boldsymbol{I} + \boldsymbol{V}_0^T\boldsymbol{M}_A\boldsymbol{V}_0 & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{T_L} \\ \boldsymbol{T_L}\ \boldsymbol{S} \end{bmatrix} = \boldsymbol{I} \tag{13}$$

Matrix $\boldsymbol{S}$ can be determined by the solution of following eigenvalue problem:

$$(\boldsymbol{P}_T - s_j\boldsymbol{N}_T)\boldsymbol{w}_{Tj} = \boldsymbol{0} \tag{14}$$

The transformation matrix and then the modal matrix can be figured out from equations (12) and (13).

In conclusion, the method of modal synthesis provides modal and spectral parameters determination of the resulting structure that consist of the original structure interpreted as a proportionally damped system and the modifying substructure described by its modal parameters (Slavík et al., 1997 and Braun et al., 1950 and Musil et al., 2014).

3. Dynamic parameters modification of cantilever beam

The above mentioned modal synthesis method can be used to modify modal and spectral properties of beam structures with added vibroinsulating layers to appropriate locations. This situation can be explained on a simple steel beam with an added beam with aluminium foam properties. The presented method can be automated and used for the parameter optimization of vibroisolating layers (orientation, geometry, material properties, etc...). The schematic representation of the optimizing position a and thickness h of the vibroisolating layer with respect to the maximum ratio of damping ξ in the second Eigen mode is shown in Fig. 2.

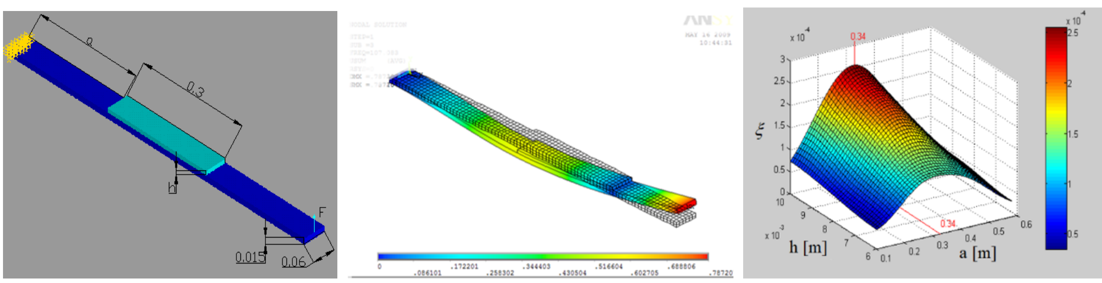
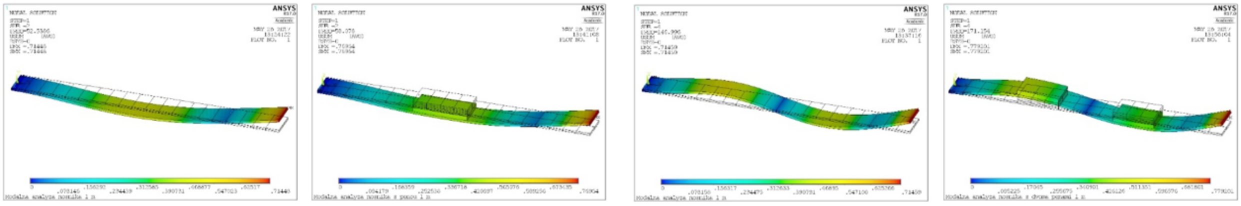


Fig. 2: Optimisation of position "a" and thickness "h" of the vibro-isolating layer with respect to the maximum ratio of damping "ξ"

It is possible to achieve the desired damping of some Eigen modes by choosing the appropriate position and thickness of the vibroisolating layer (Kirch, 1993). Therefore, it is possible to determine how effective the chosen parameters of the vibroisolating layer are without any time-consuming calculations.

4. Structural dynamic modification of the beam by several aluminum foam substructures

This chapter introduce changes of natural frequencies of a cantilever beam by adding the aluminium foam layers to appropriate locations. In this chapter there are included modal analysis results of the final element beam model modified by models of substructures. Mass properties of the substructure do not markedly influent the resulting mass matrix while its stiffness parameters significantly changed the stiffness matrix of the resulting system. Therefore, natural frequencies values, which depends mainly on mass and stiffness matrices, can be increased by adding these substructures to anti-node locations. The following picture illustrates the increasing of the second and the third natural frequency of the cantilever beam considering bending oscillation.



52.539 Hz 58.078 Hz 147.00 Hz 171.15 Hz

Fig.3: Modal analysis of cantilever beam and its aluminum layer modifications

It can be seen from the Fig. 3, that it is possible to effectively modify natural frequencies using relatively small, cheap and light additional components.

5. Conclusion

This work presented a method for a desired modification of dynamic parameters of structures by additional components using the modal synthesis. This methodology combines the structural and modal parameters of joined structures to determine dynamic parameters of the resulting system. This approach can be automated and used to design suitable additional components and to determine their locations with respect to the ratio of damping of competent modes as was shown in the third chapter on the modified beam example. This work also included the example of the effective changing of natural frequencies of the structure by adding stiffening substructures with suitable properties to appropriate locations.

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