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CONTROL OF THE NON-STATIONARY GYROSCOPIC SYSTEM IN THE TARGET TRACKING PROCESS

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Abstract: *The paper presents a method of controlling a gyroscope system constituting a drive in observation and tracking heads and in the coordinator of the target of self-guiding flying objects. In many cases, the mentioned system becomes non-stationary due to the changing parameters such as the speed of own rotation (fast) or friction in the bearings of the gyroscope frames. Classical controllers do not provide sufficient control accuracy, thus the flying object will not be able to intercept the moving target. The modified linear–quadratic regulator described previously is a solution to this problem. This article shows the effectiveness of this regulator in comparison with the optimal regulator of PD. Some results of the research are presented in a graphical form.*

Keywords: optimal regulation, modified LQR, target tracking, gyroscopic system

1. Introduction

Gyroscopic systems (GS) are still widely used in the systems of observation and tracking of the target located on the board of moving objects (Gapiński et al., 2014; Gapiński and Szmidt, 2017). As high precision of action is required from them, the stabilizing and tracking controls should be very carefully selected. The classical method of controlling the optimal gyroscope axis movement with a square quality indicator turns out to be unsatisfactory in the case of influence of external interference on GS in the form of kinematic substrate interaction, as well as the changing in time parameters of the gyroscope itself (increase or decrease in the own speed, friction in frame bearings, imbalance of the rotor, lack of coinciding of the center of mass with the center of rotation of the frames, etc.). The effect of this type of disturbance may be the non-provision of interception of the moving target to the flying object (Grzyb and Stefanski, 2017; Krzysztofik and Koruba, 2014). In such cases, the modified Linear–quadratic regulator (LQR) method should be used to control of GS.

In the paper (Koruba and Krzysztofik, 2017) an example of using this method for a non-linear gyroscope with constant parameters is presented. In this paper, the modified LQR method for controlling the non-stationary gyroscopic system (its parameters are variable over time) has been used.

2. Model of dynamics and control of the gyroscopic system

In the classical LQR method we assume linear equations of the system and state matrix A with fixed parameters. In the modified LQR method, matrix A is replaced with Jacobian determined on the basis of the nonlinear equations of motion.

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Non-linear movement equations of controlled gyroscopic system have the following form (Krzysztofik et al., 2017):

$$\frac{d^2\vartheta_g}{dt^2} = \left\{ U_b - \eta_b \frac{d\vartheta_g}{dt} - 0.5 J_{gk} \left(\frac{d\psi_g}{dt} \right)^2 \sin 2\vartheta_g - J_{go} n_g \frac{d\psi_g}{dt} \cos\vartheta_g \right\} / J_{gk} \tag{1}$$

$$\frac{d^2\psi_g}{dt^2} = \left\{ U_c - \eta_c \frac{d\psi_g}{dt} + J_{gk} \frac{d\psi_g}{dt}\frac{d\vartheta_g}{dt} \sin 2\vartheta_g + J_{go} n_g \frac{d\vartheta_g}{dt} \cos\vartheta_g \right\} / J_{gk} \cos^2\vartheta_g \tag{2}$$

where: ϑ_g, ψ_g – angles of deviation of internal and external frames of the gyroscope (angles determining the position of the gyroscope axis in space); $\dot{\vartheta}_g, \dot{\psi}_g$ – angular velocities of internal and external deviations of the gyroscope; J_{go}, J_{gk} – longitudinal and transverse moment of inertia of the rotor; n_g – speed of own rotations of the rotor; η_b, η_c – coefficients of damping in the suspension bearings.

And U_b and U_c controls are presented in the following form:

$$\begin{bmatrix} U_b \\ U_c \end{bmatrix} = -K \begin{bmatrix} \vartheta_g - \varepsilon \\ \dot{\vartheta}_g - \dot{\varepsilon} \\ \psi_g - \sigma \\ \dot{\psi}_g - \dot{\sigma} \end{bmatrix} \tag{3}$$

where: ε, σ – angles determining the location of the target observation line; $\dot{\varepsilon}, \dot{\sigma}$ – angular velocities defining the position of the target observation line.

Using the Matlab function, the matrix of **K** reinforcements appearing in equation (3) has the form (Lewis et al., 2012; Tewari, 2002):

$$\mathbf{K} = lqr(\mathbf{J}, \mathbf{B}, \mathbf{Q}, \mathbf{R}) \tag{4}$$

Matrix **J** constituting the argument of *lqr* function is a Jacobian of GS with the following components:

$J_{11} = 0,\ J_{12} = 1,\ J_{13} = 0,\ J_{14} = 0$

$J_{21} = \left(-J_{gk}\dot{\psi}_g^2 \cos 2\vartheta_g + J_{go} n_g \dot{\psi} \sin\vartheta_g \right) / J_{gk}$

$J_{22} = -\eta_b / J_{gk},\ J_{23} = 0,\ J_{24} = \left(-J_{gk}\dot{\psi}_g \sin 2\vartheta_g - J_{go} n_g \cos\vartheta_g \right) / J_{gk}$

$J_{31} = 0,\ J_{32} = 0,\ J_{33} = 0,\ J_{34} = 1$

$J_{41} = 2\sin\vartheta_g \left(-\eta_c \dot{\psi}_g + J_{gk} \dot{\vartheta}_g \dot{\psi}_g \sin 2\vartheta_g + J_{go} n_g \dot{\vartheta}_g \cos\vartheta_g \right) / J_{gk} \cos^3\vartheta_g +$
$+ \left(2J_{gk} \dot{\vartheta}_g \dot{\psi}_g \cos 2\vartheta_g - J_{go} n_g \dot{\vartheta}_g \sin\vartheta_g \right) / J_{gk} \cos^2\vartheta_g$

$J_{42} = \left(J_{gk} \dot{\psi}_g \sin 2\vartheta_g + J_{go} n_g \cos\vartheta_g \right) / J_{gk} \cos^2\vartheta_g,\ J_{43} = 0,\ J_{44} = \left(-\eta_c + J_{gk} \dot{\vartheta}_g \sin 2\vartheta_g \right) / J_{gk} \cos^2\vartheta_g$

B is matrix of control, **Q** is a positive semi-definite square, symmetric matrix called the state weighting matrix; **R** is a positive definite square, symmetric matrix called the control cost matrix.

3. Numerical example and obtained results

Similarly as in the paper (Koruba and Krzysztofik, 2017), we will consider the tracking of a moving point in space by the target coordinator placed on Earth. In the following simulation tests, the following parameter values were adopted:

1) Gyroscopic system parameters:

a) longitudinal and transverse moment of inertia of the rotor:

$J_{go} = 5 \cdot 10^{-4}$ kgm^2, $J_{gk} = 2.5 \cdot 10^{-4}$ kgm^2, $c_b = c_c = 1/J_{gk}$

b) speed of own rotations of the rotor:

$n_g = 2200 \cdot t^2 + 50$ for $t \in \langle 0{,}0.5 \rangle$ (the speed increases from 50 to 600 rad/s in 0.5 s),

$n_g = 2200 \cdot (t - 0.5)^2 + 50$ for $t \in \langle 0{,}0.5 \rangle$ (the speed decreases from 600 to 50 rad/s in 0.5 s),

c) damping in suspension bearings:

$\eta_b = \eta_c = 0.16t^2 + 0.01$ for $t \in \langle 0{,}0.5 \rangle$ (damping increases from 0.01 to 0.05 Nms in 0.5 s),

2) Program movement of a point (target) in space:

$\varepsilon = a + 0.2\omega t^2$, $\sigma = b + \omega t$ where: $a = 0.1$, $b = -0.2$, $\omega = 1.5$

3) LQR regulator parameters:

B = [0 0; cb 0; 0 0;0 cc]; Q = [1000 0 0 0; 0 10 0 0; 0 0 1000 0; 0 0 0 10]; R = [0.5 0; 0 0.5]

4) Optimal parameters of PD regulator: $k_b = 5.5$; $k_c = \frac{1}{2}\sqrt{2 + 4k_b}$, $h_g = \sqrt{2 + 4k_b}$.

The studies were conducted with an integration step amounting to $dt = 0.00001$ (Baranowski, 2013). Some results of the GS control simulation in the target tracking process are presented and compared using the optimal PD regulator and the modified LQR regulator. The graphs shown in Figs. 1-3 show that the control using the modified LQR method using Jacobian **J** works more correctly than with the optimal PD regulator – the axis of the gyroscopic system accurately reproduces the motion set along the observation line of the moving point in space.

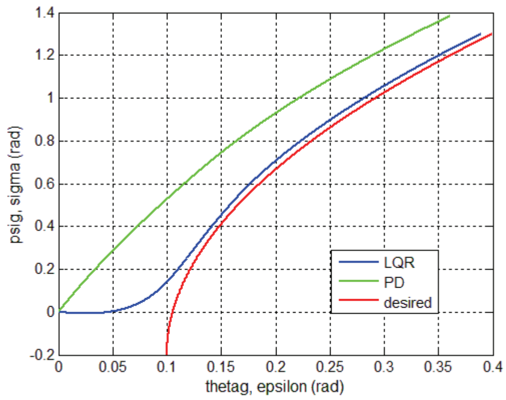

Fig. 1: Trajectories of the performed and desired motion with the use optimal PD and modified LQR for increasing rotor speed

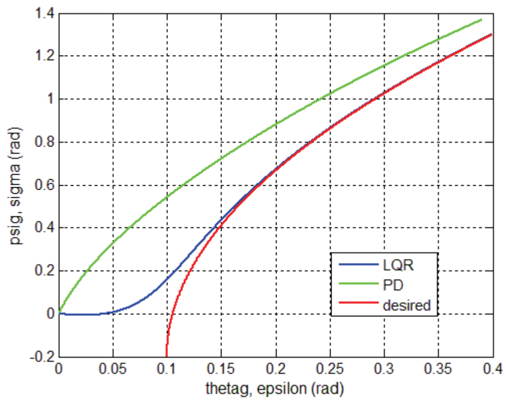

Fig. 2: Trajectories of the performed and desired motion with the use optimal PD and modified LQR for decreasing rotor speed

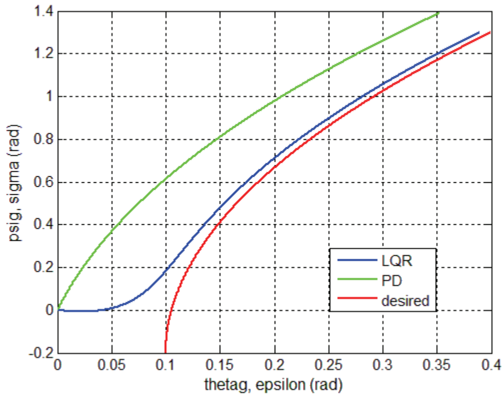

Fig. 3: Trajectories of the performed and desired motion with the use optimal PD and modified LQR for changing damping in suspension bearings

4. Conclusions

The example of controlling of the non-stationary gyroscopic system with the modified LQR method during the tracking of the target presented in this paper, allows to conclude that it is more effective than using the optimal classic PD regulator. The example analyzed in this article shows that tracking of a moving target by GS is possible even with a significant change in the speed of own rotation and changes in damping in gyroscope suspension bearings. It should be emphasized that the results of the research included in this paper (similarly as in the paper (Koruba and Krzysztofik, 2017) showed a significant improvement in the precision of the gyroscopic system control (about 10%), and this is decisive in reaching the target in missile homing systems using this type of GS. Further research will also concern the lack of knowledge of initial conditions and incomplete measurement data in gyroscopic observation and tracking systems.

References

Baranowski, L. (2013) Effect of the mathematical model and integration step on the accuracy of the results of computation of artillery projectile flight parameters. *Bulletin of the Polish Academy of Sciences: Technical Sciences*, 61, 2, pp. 475-484.

Gapiński, D., Krzysztofik, I. and Koruba, Z. (2014) Analysis of the dynamics and control of the modified optical target seeker used in anti-aircraft rocket missiles. *Journal of Theoretical and Applied Mechanics*, 52, 3, pp. 629-639.

Gapiński, D. and Szmidt, P. (2017) The control process of a scanning and tracking IR seeker using inverse dynamics, in: *Engineering Mechanics 2017*, Brno University of Technology, Brno, pp. 326-329.

Grzyb, M. and Stefanski, K. (2017) Turbulence impact on the control of guided bomb unit, in: *Engineering Mechanics 2017*, Brno University of Technology, Brno, pp. 358-361.

Koruba, Z. and Krzysztofik, I. (2017) A control with the use of LQR modified method in the gyroscope system of target tracking, in: *Engineering Mechanics 2017*, Brno University of Technology, Brno, pp. 486-489.

Krzysztofik, I. and Koruba, Z. (2014) Mathematical Model of Movement of the Observation and Tracking Head of an Unmanned Aerial Vehicle Performing Ground Target Search and Tracking. *Journal of Applied Mathematics*, 2014, Article ID 934250, 11 pages. doi:10.1155/2014/934250

Krzysztofik, I., Takosoglu, J. and Koruba, Z. (2017) Selected methods of control of the scanning and tracking gyroscope system mounted on a combat vehicle. *Annual Reviews in Control*, 44, pp. 173-182. doi:10.1016/j.arcontrol.2016.10.003

Lewis, F. L., Vrabie, D. L. and Syrmos, V. L. (2012) *Optimal Control.* John Wiley & Sons, Hoboken.

Tewari, A. (2002) *Modern Control Design with MATLAB and SIMULINK*. John Wiley & Sons, Chichester.