24th International Conference
ENGINEERING MECHANICS 2018
Svratka, Czech Republic, May 14 –17, 2018
Paper #33, pp. 485–488, doi: 10.21495/91-8-485

NUMERICAL SIMULATION OF STOCHASTIC PROCESS AS A MODEL OF TECHNICAL OBJECT STATE CHANGES

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Abstract: *The study deals with the problems connected with a numerical simulation of the process of state changes of the analyzed technical objects. The research object is a technical system of public city bus transport. The analyzed technical objects are transport means. Transport services involve accomplishment of transport tasks along established traffic routes. An example of a model of a technical object operation as well as the major assumptions of the stochastic process, which is a mathematical model of an object state changes, have been presented. The model was built on the basis of an analysis of the state space and operation related events concerning the analyzed objects. The theory of semi-Markov processes was used for mathematical modeling of the operation and maintenance process. A numerical example of the process of a technical object state change simulation has been discussed as a tool to be used for support of the discussed process analysis.*

Keywords: numerical simulation, stochastic process, semi-Markov process, operational state

1. Introduction

The research object is generally understood as a system of technical objects operation and maintenance. Operation of such a system involves performing some processes. A real system of city bus transport service of an urban agglomeration was chosen to be the research object. The study is based on an analysis of this system. The main goal of the system operation is to provide passengers with safe transport services, in a given quantitative and territorial scope. Transport means are the analyzed technical objects. Transport tasks involve carrying passengers along established traffic routes (Woropay, Knopik and Landowski, 2001).

Buses of different types and makes are used in the research object. Services are provided for several transport routes. The choice of vehicles to be used for provision of transport services for particular routes is an important issue to be addressed by decision makers. The work of (Landowski et al., 2016) includes, among others, an example of a method for choosing a technical object to perform a transport task. This method is based on an analysis of a homogenous Markov process to be used as a model of the operation and maintenance process, whereas, the work of (Landowski et al., 2017) presents economic aspects connected with a purchase of a vehicle to be used in the system of a city public transport.

This study presents an example of a semi-Markov process application for mathematical modeling and analysis of the process of a city bus transport system operation and maintenance. Determination of the values of indexes which characterize the analyzed process is performed with the use of a computer simulation of a semi-Markov process which is a mathematical model of a technical object operation and maintenance. The presented simplified numerical simulation of the operation and maintenance process applies to the problems connected with matching transport means to the task they are supposed to perform on a given traffic route.

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The problems of numerical simulation of the vehicle operational state changes are presented, among others, in the works of (Landowski and Muślewski, 2017, Landowski, Pająk, Żółtowski and Muślewski, 2017).

Sets of source data needed to make the assumptions of the model and its initial verification have been obtained from tests performed in a real research object by the method of passive experiments. Due to a limited size of the study only the basic assumptions of this method have been discussed.

Analytical determination of characteristics of the operation process, whose model is the described stochastic process, is a complex issue. Literature includes mostly analyses of models with small numbers of operational states and with limited possibilities of transitions between particular states. It appears that a numerical simulation of the process of operational state changes and determination of values of selected characteristics, can be useful in analyzing the technical object operation and maintenance process.

2. Mathematical model

A natural mathematical model of the operation and maintenance process of many categories of technical objects is random X(t) process with finite space of S states and a set of parameters R+ (subset of real numbers ≥ 0) (Buslenko, Kalashnikov and Kovalenko, 1979, Grabski, 1982). Homogeneous stochastic processes, including Markov and semi-Markov processes, are commonly used for operational state modeling (Baykal-Gürsoy, 2010, Grabski, 1982, Knopik and Migawa, 2017, Woropay, Knopik and Landowski, 2001).

Stochastic process $\{X_t, t \in R+\}$ with a finite space of S = {1, 2, 3, ..., k} states is a mathematical model of the process of a technical object operational state changes. The analyzed stochastic process $\{X_t, t \in R+\}$ was assumed to be a semi-Markov process.

In many studies, including (Grabski, 1982), one can find general properties of semi-Markov processes and possibilities of their application for an analysis of technical object operation.

T_{ij} stands for a random variable denoting the duration time of state $i \in S$ of the process $\{X_t, t \in R+\}$ when the next state is state $j \in S$. Distribution of random variable T_{ij} is defined by distribution function $F_{ij}(t)$. From the point of view of the study goal, it is convenient to unequivocally describe the analyzed semi-Markov process through identification of elements $(p, P, F(t))$ which denote, respectively: stochastic vector of the initial distribution $p = [p_i : i \in S]$, $p_i = P\{X(0) = i\}$, $i \in S = \{1, 2, 3, ..., k\}$, $\sum_{i=1}^{k} p_i = 1$, transition probability matrix of Markov chain $P = [p_{ij} : i, j \in S]$, $p_{ij} = P\{X(\tau_{n+1}) = j / X(\tau_n) = i\}$ embedded in process $\{X_t, t \in R+\}$ and the matrix of random variable distribution functions T_{ij}, $i, j \in S$ denoting duration time of state i of process $\{X_t, t \in R+\}$ if the successive state is state j $F(t) = [F_{ij}(t) : i, j \in S]$, $F_{ij}(t) = P\{T_{ij} < t\} = P\{\tau_{n+1} - \tau_n < t / X(\tau_{n+1}) = j, X(\tau_n) = i\}$, where: $0 = \tau_0 < \tau_1 < \tau_2 < \tau_3 ... < \tau_n <..$ - random variables denoting time moments in which changes in states of process X(t), $j, i, i_{n-1}, i_{n-2}, ... i_1, i_0 \in S = \{1, 2, 3, ..., k\}$ occur, n - number of the step in which change in the state of process X(t), occurs, $n \in N$, N– a set of natural numbers.

The definition of semi-Markov process implies the following dependencies:

$$Q_{ij}(t) = p_{ij} F_{ij}(t), \quad i, j \in S, \ t \in R+ \tag{1}$$

$$\sum_{j \in S} Q_{ij}(t) = F_i(t) \text{ for } i \in S, \ t \in R+. \tag{2}$$

Matrix $Q(t) = [Q_{ij}(t) : i, j \in S]$ is referred to as the semi-Markov process core. Function $F_i(t)$ is a distribution function of state $i \in S$ duration time. Random variable denoting duration time of state $i \in S$ with a distribution defined by distribution function $F_i(t)$, is marked by symbol T_i.

The basic property of semi-Markov process $\{X_t, t \in R+\}$ can be defined as follows: for a given state of the process in time τ_n the duration time of an actual state and the state reached in time τ_{n+1} do not depend stochastically on the process states in times τ_0, τ_1, ..., τ_{n-1} or on duration times of the previous states, which can be expressed as follows:

$$P\{X(\tau_{n+1}) = j, \tau_{n+1} - \tau_n < t / X(\tau_n) = i\} = \\ = P\{X(\tau_{n+1}) = j, \tau_{n+1} - \tau_n < t / X(\tau_n) = i, X(\tau_{n-1}) = i_{n-1}, ..., X(\tau_1) = i_1, X(\tau_0) = i_0\}, \quad (3)$$

3. Numerical simulation of the process of operation and maintenance - calculation example

With the described above assumptions, simulation of the operation process of a single technical object involves numerical simulation of the described stochastic process implementation, being a model of the process of a technical object operational state changes. Algorithms have been developed to enable numerical simulation of the stochastic process. Data necessary to define the described stochastic process was needed to perform the simulation. Values of selected indexes, enabling an analysis of the modeled process of a technical object state changes, were determined during the simulation.

The study does not include a full set of input data necessary to simulate the analyzed stochastic process or the method for estimation of the model parameters values determined on the basis of results of tests conducted in a real system. Due to the character of this study and a wide scope of obtained results only the results of selected simulation experiments are presented. The presented simulation results are of illustrative character.

Numerical simulation of the operation process by means of the developed computer simulation program allows to determine the values of a set of indexes for different input quantities. Input quantities for the analyzed time interval, determined during the simulation, are the values of selected operational indexes such as: total time of repairs, costs of repairs, incomes from transport services, transport task accomplishment, number of repairs and others.

A simplified model of the process of public bus transport system operation presented further in the study illustrates the possibilities of using semi-Markov processes and a computer simulation for an analysis and control of a technical object operation process.

Three operational states of a bus operation have been analyzed. State S_1 - transport task is being accomplished. State S_2 - the technical object is being serviced by a mobile technical emergency service. State S_3 - the technical object is being serviced inside the transport system. Each technical object is assumed to be in only one of the identified operational states, in time t, thus forming a set of the process states.

Being in a given state involves bearing costs or gaining income, in unit of time. Change in the operational state may result in bearing costs or obtaining income. Operational states are characterized by distributions of their duration times. Duration times of states, sequence of successive states, incomes and costs depend, among others, on the type of transport means.

It was assumed that being in state S_1 involves obtaining income, whereas, the remaining states generate costs for the system which operates the technical objects.

In the considered model, the space of S states of X(t) process consists of three states S = {i : I = 1,2,3}. States i of the analyzed stochastic process correspond to the identified operational states S_i of a bus.

In the considered example, the efficiency of transport task accomplishment by the studied transport means, on the established traffic routes, has been analyzed. The values of the model parameters are estimated for four types of buses used in the discussed system. The experiments involved a numerical simulation of the transport process operation with the use of the analyzed vehicles.

Selected results of the simulation experiments are presented in Tab. 1. The results presented in Tab. 1 include mean values of the income obtained by the system from providing transport services by one vehicle, on an analyzed route, for twenty four hours.

Tab. 1: Income (PLN) obtained for twenty four hours from providing transport services by the analyzed vehicles.

Vehicle code	Mean value
Bus of type X1	245,61
Bus of type X2	268,23
Bus of type X3	196,81
Bus of type X4	182,61

4. Conclusions

The presented numerical simulation of the process of a technical object operational state changes indicates a possibility of using the developed method and a computer simulation program as tools to be used to support decision makers in the analyzed technical system.

Numerical simulation of a semi-Markov process which is a model of the process of a technical object state changes allows to determine the process characteristics which are difficult to determine in an analytical way. In particular, it applies to models with numerous spaces of states with a big number of non-zero values of probabilities of transitions between the states and non-homogenous processes. The set of indexes that can be determined includes subsets of the following indexes: availability, repair time, efficiency of transport services, costs and others.

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