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VALIDATION OF HOMOGENIZED MODEL OF THE FLUID-SATURATED PIEZOELECTRIC MEDIA

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Abstract: *This contribution is devoted to validation of the homogenized model of porous piezoelectric media. We consider the solid piezoelectric skeleton with embedded electrically inert fluid inclusions and the two conducting electrodes in which different electric potentials are prescribed. The homogenized model provides the macroscopic results which are together with characteristic responses of the microstructure used for reconstruction of the strain and electric fields at the level of heterogeneities. These reconstructed fields are compared to the responses of the reference model.*

Keywords: piezoeletric materials, porous media, homogenization, multiscale modelling

1. Introduction

Piezoelectric materials have many applications in modern technologies, such as mechatronics, electronics, etc. They can be found also in natural structures in the form of crystals. They are often used in smart structures as actuating parts for self-monitoring or self-control, e.g. in the aerospace industry to control vibrations, or acoustic radiation of thin flexible constructions. There are different methods for mathematical modelling of heterogeneous media consisting of piezoeletric components such as Mori–Tanaka and self consistent upscaling schemes,see Ayuso (2017), or the periodic homogenization approaches based on the two-scale asymptotic expansions, on the two-scale convergence, see Allaire (1992), or on periodic unfolding method , Cioranescu (2008).

The aim of this contribution is to show that the presented homogenization approach for modeling porous piezoelectric media, see Rohan (2018); Miara (2015), provides reliable results with lower computational demands in comparison to the direct numerical simulation. The homogenization procedure leads to the decoupled problems at two levels. We solve a problem for the unknown displacement and pressure fields at the macroscopic level and several subproblems to find the local microstructural responses. The local responses and the global fields are used to reconstruct the solution, i.e. the displacements and the electric potential and consequently also the strain and electric fields at the microscopic level for a given size of the heterogeneities. These reconstructed fields are comparable to those computed for the non-homogenized medium.

2. Homogenization of fluid-saturated piezoelectric medium

We consider the piezo-poroelastic medium with a periodic lattice occupying an open bounded domain $\Omega \subset \mathbb{R}^3$ which can be decomposed into the piezoelectric matrix, Ω_m, elastic conductors, Ω_*, and fluid-saturated inclusions, Ω_c:

$$\Omega = \Omega_c \cup \Omega_m \cup \Omega_* \,, \quad \Omega_c \cap \Omega_m \cap \Omega_* = \emptyset \,, \quad \text{where } \Omega_* = \bigcup_k \Omega_*^k \,. \tag{1}$$

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In the case of quasi-static loading when inertia and viscosity effects can be neglected, the state of the solid structure is governed by the following equilibrium equations:

$$\begin{aligned} -\nabla \cdot \boldsymbol{\sigma}(\boldsymbol{u}, \varphi) &= \boldsymbol{f} \;, \quad \text{in } \Omega_{m*} \;, \\ -\nabla \cdot \vec{D}(\boldsymbol{u}, \varphi) &= q_E \;, \quad \text{in } \Omega_m \;, \end{aligned} \tag{2}$$

where $\boldsymbol{u}$ is the displacement, φ is the electric potential, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\vec{D}$ is the electric displacement, $\boldsymbol{f}$ is the the volume-force and q_E is the volume electric charge. The Cauchy stress tensor and the electric displacement are determined by the following constitutive equations:

$$\begin{aligned} \sigma_{ij}(\boldsymbol{u}, \varphi) &= A_{ijkl} e_{kl}(\boldsymbol{u}) - g_{kij} (\nabla\varphi)_k \;, \\ D_k(\boldsymbol{u}, \varphi) &= g_{kij} e_{ij}(\boldsymbol{u}) + d_{kl} (\nabla\varphi)_l \;, \end{aligned} \tag{3}$$

where $\mathbb{A} = (A_{ijkl})$ is the elasticity fourth-order symmetric tensor. The deformation is coupled with the electric field through the 3rd order tensor $\boldsymbol{g} = (g_{kij})$, $g_{kij} = g_{kji}$ and $\boldsymbol{d} = (d_{kl})$ is the permitivity tensor.

The mass conservation for the isolated inclusions yields

$$\int_{\partial\Omega_c k} \boldsymbol{u} \cdot \boldsymbol{n} \, \mathrm{dS} + \gamma p^k |\Omega_c^k| = 0 \;, \quad \forall k \in \{1, \dots, \bar{k}\} \;, \tag{4}$$

where $\boldsymbol{n}$ is the unit normal vector, γ is the fluid compressibility, p^k is the pressure in the k-th inclusion and $\Omega_c^k \subset \Omega_c$ is the domain occupied by the inclusion.

The homogenization methods based on the two scale convergence or the unfolding operator techniques, Cioranescu (2008), can be used for the asymptotic analysis of the problem stated above for $\varepsilon \to 0$, where ε is the scale parameter reflecting the size of a periodic unit. The homogenization process results in the local problems for computing the so-called characteristic responses at the microscopic level and the macroscopic model equations. Note that in order to remain the electric field bounded with $\varepsilon \to 0$, the dielectric properties (tensors $\boldsymbol{g}$, $\boldsymbol{d}$ in (3)) of the piezoelectric material must getting smaller in the right order.

The macroscopic problem is expressed in terms of the homogenized coefficients and is solved for the unknown macroscopic displacement $\boldsymbol{u}^0 \in \boldsymbol{\mathcal{U}}(\Omega)$ ($\boldsymbol{\mathcal{U}}(\Omega)$ is the admissibility set) and macroscopic pressure $p^0 \in L^2(\Omega)$:

$$\begin{aligned} \int_\Omega \boldsymbol{e}(\boldsymbol{v}^0) : \left(\mathbb{A}^H \boldsymbol{e}(\boldsymbol{u}^0) - p\boldsymbol{B}^H\right) \mathrm{dV} &= -\int_\Omega \boldsymbol{e}(\boldsymbol{v}^0) : \left(\sum_k \mathbf{H}^{H,k} \bar{\varphi}^k + \boldsymbol{S}^H \rho_E\right) \mathrm{dV} \\ &\quad + \int_\Omega \hat{\boldsymbol{f}} \cdot \boldsymbol{v}^0 \, \mathrm{dV} + \int_{\partial\Omega} \bar{\boldsymbol{h}} \cdot \boldsymbol{v}^0 \, \mathrm{dS}_x \;, \\ \int_\Omega q^0 \left(\boldsymbol{B}^H : \boldsymbol{e}(\boldsymbol{u}^0) + pM^H\right) \mathrm{dV} &= \int_\Omega q^0 \left(\sum_k Z^k \bar{\varphi}^k + R^H \rho_E\right) \mathrm{dV} \;, \end{aligned} \tag{5}$$

for all $\boldsymbol{v}^0 \in \boldsymbol{\mathcal{U}}_0(\Omega)$ and for all $q^0 \in L^2(\Omega)$. Above $\boldsymbol{e}(\boldsymbol{u}) = \frac{1}{2}(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T)$ is the strain tensor, $\hat{\boldsymbol{f}}$ are the volume forces, $\bar{\boldsymbol{h}}$ are the surface tractions, ρ_E is the surface charge and $\bar{\varphi}^k$ is a given potential in the k-th conductor.

The homogenized coefficients $\mathbb{A}^H$, $\boldsymbol{B}^H$, $\mathbf{H}^{H,k}$, $\boldsymbol{S}^H$, M^H, Z^k and R^H are computed using the characteristic responses obtained by solving several decoupled problems at the microscopic level. Using the macroscopic solution ($\boldsymbol{u}^0(x)$, $p^0(x)$) of (5) and the local results ($\boldsymbol{\omega}^{ij}(y)$, $\boldsymbol{\omega}^P(y)$, $\boldsymbol{\omega}^\rho(y)$, $\hat{\boldsymbol{\omega}}^k(y)$, $\hat{\eta}^{ij}(y)$, $\hat{\eta}^P(y)$, $\hat{\eta}^\rho(y)$, $\hat{\eta}^k(y)$), we are able to reconstruct the strains and the electric field for a finite scaling parameter ε^0 at the level of the heterogeneities. The local gradients can be expressed as:

$$\begin{aligned} \boldsymbol{e}^{\mathrm{mic}}(x, y) &= \boldsymbol{e}_x(\boldsymbol{u}^0) + \boldsymbol{e}_y(\boldsymbol{\omega}^{ij}(y)) e_{ij}^x(\boldsymbol{u}^0) - p^0(x) \boldsymbol{e}_y(\boldsymbol{\omega}^P(y)) + \boldsymbol{e}_y(\boldsymbol{\omega}^\rho(y))\rho_E + \sum_k \boldsymbol{e}_y(\hat{\boldsymbol{\omega}}^k(y))\bar{\varphi}^k \;, \\ \vec{\boldsymbol{E}}^{\mathrm{mic}}(x, y) &= \frac{1}{\varepsilon^0} \left(\nabla_y \hat{\eta}^{ij}(y) e_{ij}^x(\boldsymbol{u}^0) - p^0(x) \nabla_y \hat{\eta}^P(y) + \nabla_y \hat{\eta}^\rho(y)\rho_E + \sum_k \nabla_y \hat{\varphi}^k(y) \bar{\varphi}^k\right) . \end{aligned} \tag{6}$$

3. Model validation

The homogenized model of piezoelectric porous media is validated against results obtained by the direct numerical simulation of the periodic structure. The reference model is built up by copies of the periodic unit cell for a given finite ε^0. We consider a block sample made of barium-titanite piezoelectric matrix in which the fluid inclusions and the two networks of metallic conductors are embedded, see Fig. 1. The boundary conditions applied to this sample are depicted in Fig. 1 right, in y direction the periodic condition is employed. No surface electric charge, no volume forces and surface tractions are considered in our validation test. The deformation of the sample is induced by the piezoelectric effect due to the prescribed electric potentials $\bar{\varphi}^1$, $\bar{\varphi}^2$. The responses of the reference and homogenized model are compared in Fig. 2, where the magnitudes of the strain field and the electric field are depicted. The graphs in Fig. 2 bottom show the relative difference of the quantities along a given line. The both models are solved by means of the finite element method in *SfePy* package, see Cimrman (2014).

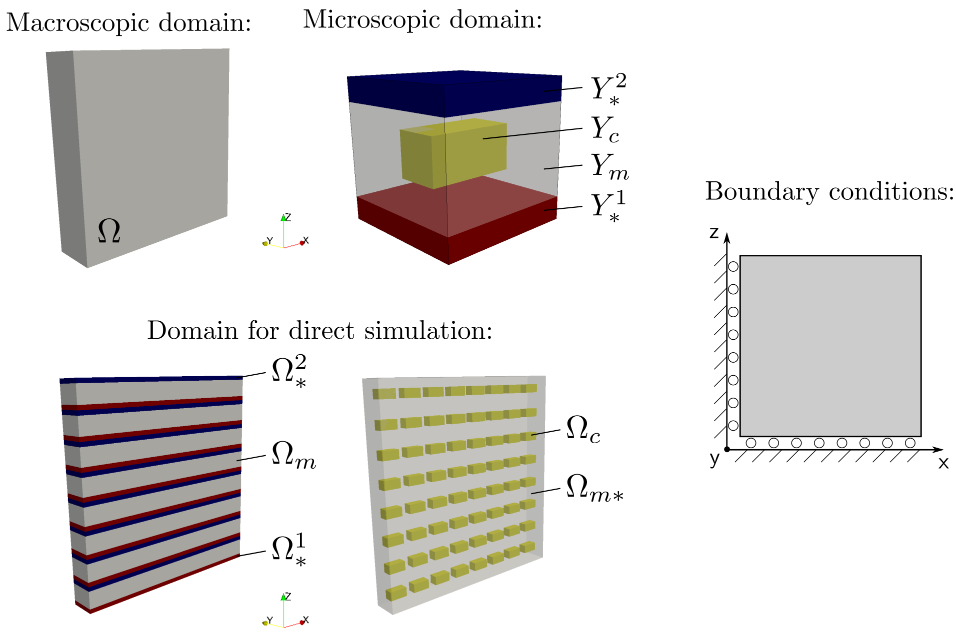

Fig. 1: Computational domains and boundary conditions: top – macroscopic domain Ω (left) and microscopic reference cell Y (right); bottom – domain use in the reference model; right — boundary conditions applied to the sample.

4. Conclusion

The homogenized model of piezoeletric fluid-saturated porous media was validated using the direct numerical finite element simulation of a given heterogeneous structure. In the validation test, the decoupled microscopic problems with 741 degrees of freedom (DOFs) are solved several times to compute all the corrector functions. This procedure is followed by the solution of the macroscopic problem with 577 DOFs and by reconstruction of the microscopic fields. The computational time of the homogenized model, including reconstructions at the microscopic level, is about 15 times faster than the solution of the reference model which has approximately 4.5×10^5 DOFs. It is the significant reduction of computational cost. The results of the homogenized and the reference model are in a good agreement except the parts close to the domain boundary where the periodicity assumption applied in the homogenized model is not satisfied.

Acknowledgments

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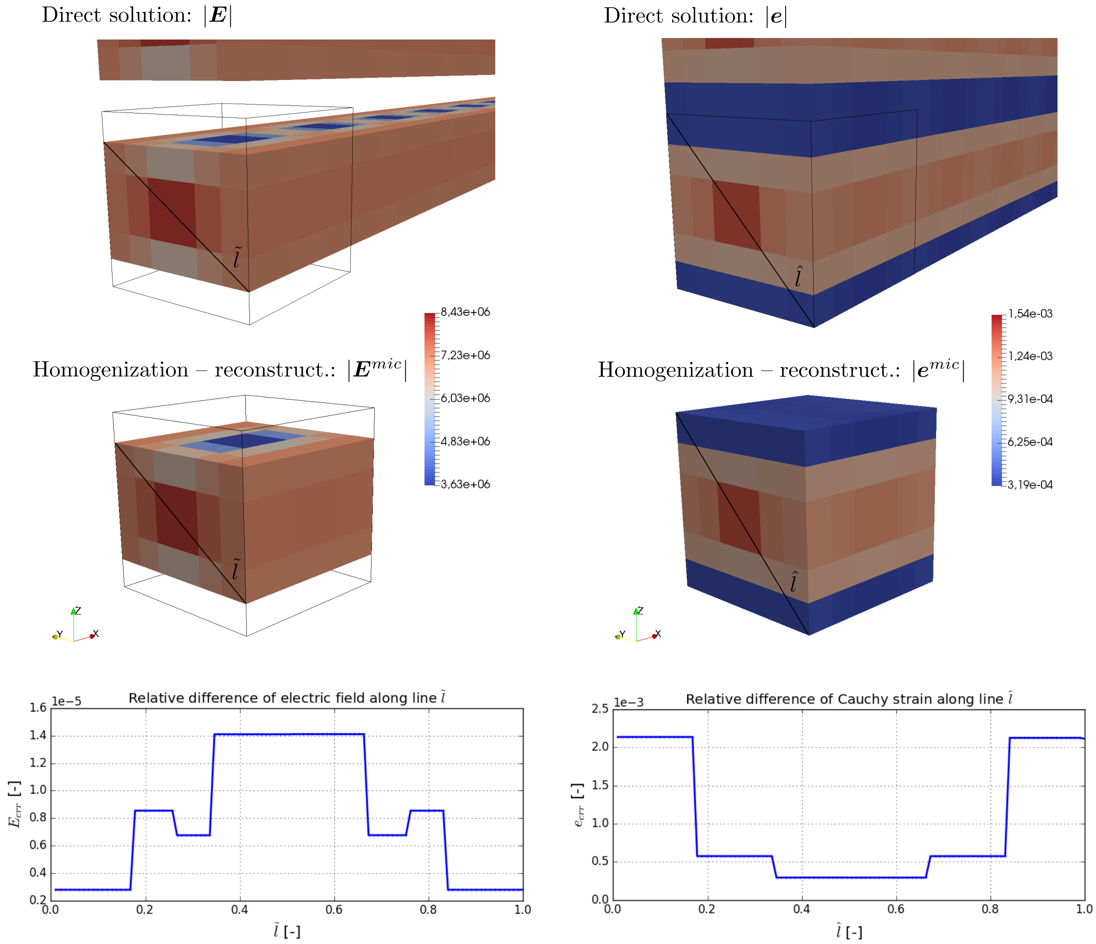

Fig. 2: Responses of the reference (top) and homogenized (middle) model and the relative difference of the results (bottom); left – strain field magnitude, right – electric field magnitude.

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