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NUMERICAL SIMULATIONS OF AN INFLUENCE OF THE PARAMETERS OF LOADING HEADS ON THE LOADING CAPACITY OF A DAMAGED COLUMN SUBJECTED TO A SPECIFIC LOAD

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Abstract: *The studies on an influence of the parameters of the loading heads on the loading capacity of the cracked column subjected the specific load realized with the circular elements of heads are presented in this paper. The crack is simulated by means of the rotational spring. The boundary problem is formulated with the use of the minimum total potential energy principle on the basis of which the differential equations as well as natural boundary conditions are obtained. The main scope of investigations is to control the loading capacity of the cracked system by means of proper selection of the parameters of the loading heads.*

Keywords: pecific load, crack, static stability criterion, column

1. Introduction

The specific load, considered in this paper, was introduced into literature by Tomski (Tomski et al., 1994) and can be realized by means of the loading structures (heads). The outlines of the heads have a shape in the form of: circular (Tomski et al., 2004), parabolic (Tomski et al., 2011) or linear elements (Tomski et al., 1994). The specific load can be realized as: the generalized load with a force directed towards a negative or positive pole (Tomski et al., 1994), the load by a follower force directed towards a negative or positive pole (Tomski et al., 1998). The mentioned pole is a point localized below the loaded end of the column on its undeformed axis. A line of action of the external force is created by the line that connects the pole and the loaded end of the column. The crack presence affects and changes the static and dynamic behavior of system. Finally, the crack growth leads to the destruction of each structure. An early detection of the crack and the repair or improvement of the host structure due to the crack presence is a basic task for engineers. In the literature cracks are divided into two types. The first one presents the linear problems where the cracks remain always open while the second type focuses on non-linear problems where the crack opens and closes in time. In the numerical simulations, the different approach to cracks can be observed. Generally, cracks are simulated with the use of reduced cross sectional area, rotational springs or complex mathematical functions. The results of the studies on cracks are presented in the following papers (Cekus et al., 2016, Sokół et al., 2016, Zamorska et al., 2015). In the scientific papers a reduction of the loading capacity of the slender systems as a result of crack presence is discussed. Moreover, the presented systems are loaded by conservative Euler's load. In this study the specific load is used as a source of external load acting on the cracked supporting system. The main aim of the theoretical and numerical studies is to find such a combination of the parameters of the loading heads at which an influence of the crack presence on loading capacity will be reduced or removed.

2. The boundary problem formulation

In Figure 1 the investigated system is presented. The external load is realized by means of the loading heads with circular outline. The presence of the crack divides a column into two elements. During

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numerical calculations the crack is always open and is simulated by the rotational spring of stiffness C which can be calculated with consideration of (Ostachowicz et al., 1991). The continuity of transversal and longitudinal displacements as well as bending moments and deflection angles is satisfied by natural boundary conditions in the point of crack presence. The total length of the structure is $l=l_1+l_2$. The loading head of radius R can move smoothly in the vertical direction. The radius R has a center in the point localized below the loaded end of the column on its undeformed axis through which passes the line of P force action (pole point). The radius of the receiving head is r and the distance between the end of the column and the contact point of both heads is l_0. The physical and geometrical features of the structure allows one to introduce the Bernoulli – Euler theory (Uzny et al., 2015).

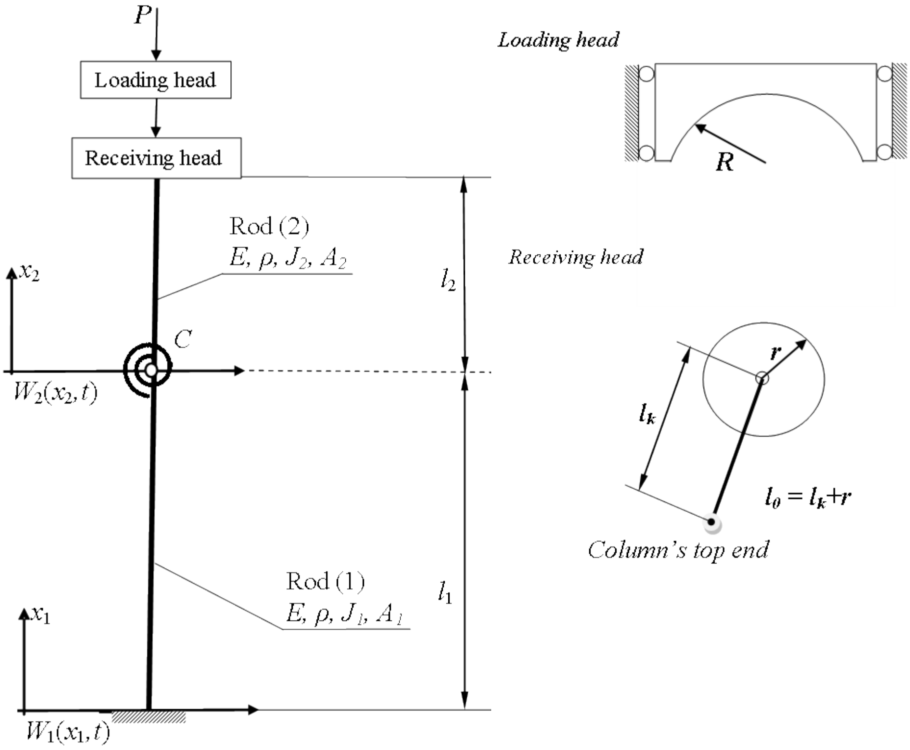

Fig. 1. The investigated column and shapes of heads

The boundary problem has been formulated on the basis of the minimum total potential energy principle (Uzny et al., 2015). An integration and variation operations performed on potential energy equation allows one to obtain the differential equations

$$EJ_i \frac{d^4 W_i(x_i)}{dx_i^4} + P \frac{d^2 W_i(x_i)}{dx_i^2} = 0 \quad i = 1,2 \tag{1}$$

and a set of geometrical (written on the basis of the geometry of the column) as well as natural boundary conditions:

$$W_1(0) = \left. \frac{dW_1(x_1)}{dx_1} \right|_{x_1=0} = 0 \qquad W_2(0) = W_1(l_1) \tag{2, 4}$$

$$-EJ_2 \left. \frac{d^2 W_2(x_2)}{dx_2^2} \right|_{x_2=0} + C \left[\left. \frac{dW_2(x_2)}{dx_2} \right|_{x_2=0} - \left. \frac{dW_1(x_1)}{dx_1} \right|^{x_1=l_1} \right] = 0 \tag{5}$$

$$EJ_1 \left. \frac{d^2 W_1(x_1)}{dx_1^2} \right|^{x_1=l_1} - C \left[\left. \frac{dW_2(x_2)}{dx_2} \right|_{x_2=0} - \left. \frac{dW_1(x_1)}{dx_1} \right|^{x_1=l_1} \right] = 0 \tag{6}$$

$$EJ_2 \frac{d^2W_2(x_2)}{dx_2^2}\Bigg|^{x_2=l_2} + P\frac{r-l_0}{R-r}\zeta = 0\,,\; EJ_2 \frac{d^3W_2(x_2)}{dx_2^3}\Bigg|^{x_2=l_2} + P\frac{1}{R-r}\zeta = 0 \qquad (7, 8)$$

$$\zeta = \frac{dW_2(x_2)}{dx_2}\Bigg|^{x_2=l_2} (R-l_0) - W_2(l_2) \qquad (9)$$

The results of numerical simulations are plotted in the non-dimensional form on the basis of the parameters defined as:

$$p_B = \frac{Pl^2}{EJ_1},\; c = \frac{Cl}{EJ_1},\; k_A = \frac{R}{l},\; k_B = \frac{r}{R},\; k_C = \frac{l_0}{R},\; \mu = \frac{EJ_2}{EJ_1},\; d_2 = \frac{l_2}{l} \qquad (10a\text{-}g)$$

The results presented in this paper have been carried for $\mu = 1$.

3. The results of numerical simulations

The results presented in this paragraph are plotted at rotational spring stiffness c = 1.

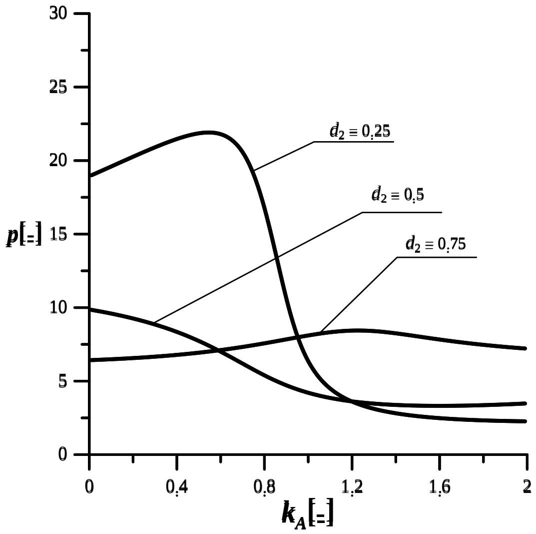

Fig. 2. Influence of k_A on the loading capacity of the column at different crack location. (c = 1, k_B = 0.25, k_C = 0.1, μ =1)

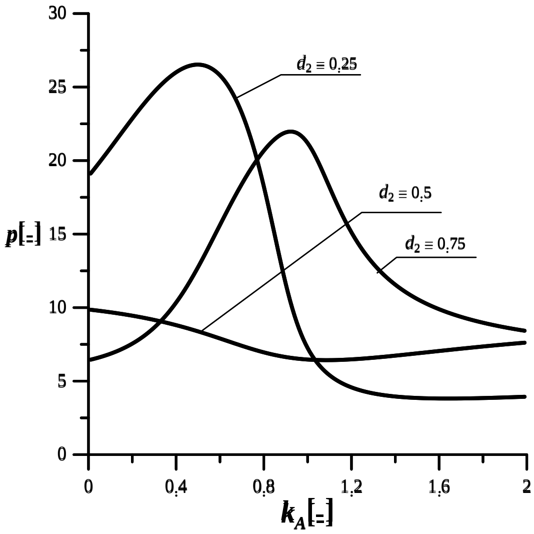

Fig. 3. Influence of k_A on the loading capacity of the column at different crack location. (c = 1, k_B = 0.50, k_C = 0.1, μ =1)

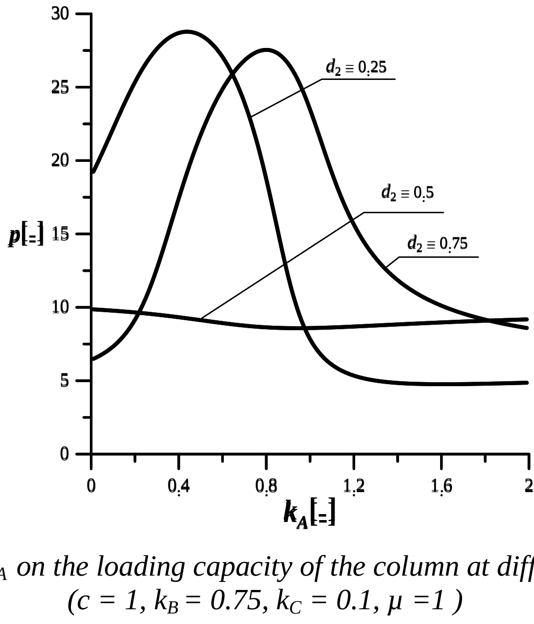

Fig. 4. Influence of k_A on the loading capacity of the column at different crack location. (c = 1, k_B = 0.75, k_C = 0.1, μ =1)

The non-dimensional radius of the loading head can vary from k_A = 0.01 up to k_A = 2. The transom length is constant - k_C = 0.1. In each of the figures, the considered defect in the form of a crack is located at different height (d_2 parameter). The variable parameter is k_B.

When the crack is located at d_2 = 0.25 from the fixed support, an increase in k_A results in initial increase of the loading capacity up to the maximum magnitude, after that the decrease can be observed. Such a tendency occurs at every presented configuration of the loading structures. It should be stated that, the maximum loading capacity point is located at smaller k_A when an increase in k_B takes place (Figures 2 – 4). When the crack is shifted to d_2 = 0.5 an increase in k_A results in reduction of the maximum external load up to the point of the lowest loading capacity, after that a small increase of the investigated parameter can be observed. At d_2 = 0.75 similar tendency to the one as at d_2 = 0.25 can be found. It can also be concluded that such a magnitude k_A can be found at which the column's capacity becomes constant. As presented the proper selection of the parameters of the loading head may lead to the increase in loading capacity of the investigated supporting structure.

4. Conclusions

In this paper the results of simulations of the loading capacity of a cracked column subjected to the specific load realized by circular elements of the heads are presented. The crack was modeled by means of the rotational spring. The main scope of the study was to find such a combination of the parameters of the loading heads at which the system will be the least sensitive to the crack presence. On the basis of the obtained results the general conclusions are as follows:

- the crack presence causes the reduction of the loading capacity of the column but the change of the parameters of the loading heads may result in decrease of sensitivity of the system to the crack,
- the area of the insensitivity strongly depends on the parameters of the loading heads and the crack size/location,
- the change of the crack location causes great differences in the loading capacity.

References

Cekus, D. Miara, M. Zamorska, I. (2016) Damage identification of a beam with a variable cross-sectional area. Journal of Applied Mathematics and Computational Mechanics, 15, 4, pp. 23-32, DOI: 10.17512/jamcm.2016.4.03.

Ostachowicz, W.M., Krawczuk, M. (1991) Analysis of the effect of cracks o the natural frequencies of cantilever beam, Journal of Sound and Vibration, 150, 2, pp.191–201.

Tomski, L. Gołębiowska Rozanow, M. Szmidla, J. (1994) Free vibration of a column loaded by a force and bending moment, Vibrations in Physical Systems, 16, Poznań – Błażejewko, pp. 317–319.

Tomski, L. Przybylski, J. Gołębiowska Rozanow, M. Szmidla, J. (1998) Vibration and stability of a cantilever column subject to a follower force passing through a fixed point, J. of Sound and Vibration, 214, 1 pp. 67–81

Tomski, L., Szmidla, J. (2004) Vibration and stability of column subjected to generalized load by a force directed towards a pole, Journal of Theoretical and Applied Mechanics, 42, 1, pp. 163–193.

Tomski, L., Uzny, S. (2011) The new realization of the specific load – free vibrations of a column, Dynamical Systems, Analytical/Numerical Methods, Stability, Bifurcation and Chaos, Łódź, pp. 301-306.

Sokół K., Uzny S. (2016) Stability of a damaged slender structure loaded by a force directed towards a positive pole, Engineering Mechanics 2016, IT AS CR, Prague, pp. 514-517.

Uzny S., Sokół K. (2015) The Bernoulli-Euler and Timoshenko Theories in the context of research on the characteristic curves of column with different boundary conditions, AIP Conference Proceedings, 1648, DOI: 10.1063/1.4913091

Uzny S., Sokół K., Kutrowski Ł. (2015) Stability of a hydraulic telescopic cylinder subjected to Euler's load, Proceedings of the 13th International Scientific Conference: Computer Aided Engineering, DOI: 10.1007/978-3-319-50938-9_60

Zamorska I., Cekus D., Miara M. (2015) Effect of crack parameters on free vibrations of the Berrnoulli-Euler beam, Journal of Applied Mathematics and Computational Mechanics. 14, 4, pp. 167-174, DOI: 10.17512/jamcm.2015.4.17.