

## A SIMPLE MODEL FOR PARAMETRIC STUDY OF PNEUMATIC-CABLE STRUCTURES

M. Biernacka<sup>\*</sup>, W. Gilewski<sup>\*\*</sup>, P. Obara<sup>\*\*\*</sup>

**Abstract:** *Pneumatic-cable structures are a new concept introduced to the engineering art at the beginning of XXI century. The present paper consists of a simple but efficient moderately thick beam finite element computational model. The parametric study is provided to define the importance of selected parameters to the static response of the beam. Pressure of the air-beam is the key parameter. Less important parameters are: height of cross-section and span of the steel beam as well as the radius of air-beam.*

**Keywords:** pneumatic structures, cable structures, beam theory

### 1. Introduction

The idea of pneumatic-cable beams is to reinforce inflated beams with cables and struts in order to improve the load bearing capacity of the system (Luchsinger et al., 2004, Pedretti, 2004 and de Laet et al., 2008). The concept was named Tensairity as an acronym for tension, air and integrity. The first Tensairity construction was built in 2002 and it was a demonstration car bridge of the span 20 m (Biernacka and Gilewski, 2018). In the coming years, the technology was continuously improved. The most famous construction is the roof of the parking garage in Montreux which was built in 2004 and shows the full potential of the Tensairity technology (Biernacka and Gilewski, 2018). Another example is skiers bridge in France with its 52 m span (Biernacka and Gilewski, 2018). Basic form of Tensairity beam consists of: an air-beam, a compression element, which is tightly connected with air-beam, two cables running in helical form around the air-beam, which are connected with compression element in the both ends (Fig. 1). In classical beam the top is in compression and the bottom – in tension. In order to reduce the weight of the beam in case of Tensairity technology the inflated tube covered by cables is proposed. The air tube stabilizes the compression element and transports the load between compression and tension elements. The mechanics of a Tensairity beam is a mix of the beam theory and the membrane theory.

The present paper is dedicated to the analysis of the importance of the pneumatic-cable beam parameters for the static response of the structure, with the use of a simple moderately thick beam finite element model.

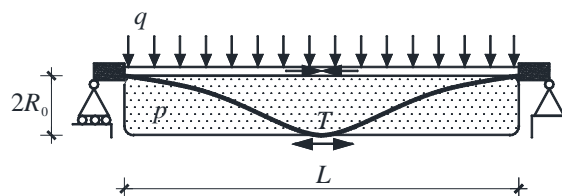


Fig. 1: Pneumatic-cable beam (Pedretti, 2004).

<sup>\*</sup> Marta Biernacka, MSc.: Warsaw University of Technology, Faculty of Civil Engineering, Al. Armii Ludowej 16, 00-637 Warsaw, PL, m.biernacka@gmail.com

<sup>\*\*</sup> Prof. Wojciech Gilewski, DSc.: Warsaw University of Technology, Faculty of Civil Engineering, Al. Armii Ludowej 16, 00-637 Warsaw, PL, w.gilewski@il.pw.edu.pl

<sup>\*\*\*</sup> Paulina Obara, PhD.: Kielce University of Technology, Faculty of Civil Engineering and Architecture, Al. Tysiąclecia PP 7, 25-314 Kielce, PL, paula@tu.kielce.pl

## 2. Beam finite element model

A simple two-noded beam finite element model with high precision physical shape functions is proposed for the analysis of the pneumatic-cable structure. Physical shape functions are obtained from the static homogeneous displacement equations of the Timoshenko beam theory and are exact for static analysis (for detailed description see Gilewski, 2013, Gilewski and Obara, 2007). The details of the finite element are the following: length –  $2a$ , cross-section –  $A$ , moment of inertia –  $J$ , Young's modulus –  $E$ , Poisson's ratio –  $\nu$ , elastic foundation coefficient –  $k_0$ , axial force –  $S$  and  $H = \frac{5EA}{12(1-\nu)}$ ,  $\gamma = \frac{EJ}{Ha^2}$ ,  $\mu = \frac{3\gamma}{1+3\gamma}$ .

Stiffness, geometric stiffness as well as elastic foundation matrices and load vector is used for the analysis on the finite element level in the following form:

$$\mathbf{K}^e = \frac{3EJ}{2a^3} \begin{bmatrix} 1-\mu & a(1-\mu) & -1+\mu & a(1-\mu) \\ a(1-\mu) & a^2\left(\frac{4}{3}-\mu\right) & -a(1-\mu) & a^2\left(\frac{2}{3}-\mu\right) \\ -1+\mu & -a(1-\mu) & 1-\mu & -a(1-\mu) \\ a(1-\mu) & a^2\left(\frac{2}{3}-\mu\right) & -a(1-\mu) & a^2\left(\frac{4}{3}-\mu\right) \end{bmatrix} \quad (1)$$

$$\mathbf{K}_g^e = \frac{S}{10a} \begin{bmatrix} 5+(1-\mu)^2 & a(1-\mu)^2 & -5-(1-\mu)^2 & a(1-\mu)^2 \\ a(1-\mu)^2 & a^2\left[\frac{5}{3}+(1-\mu)^2\right] & -a(1-\mu)^2 & a^2\left[-\frac{5}{3}+(1-\mu)^2\right] \\ -5-(1-\mu)^2 & -a(1-\mu)^2 & 5+(1-\mu)^2 & -a(1-\mu)^2 \\ a(1-\mu)^2 & a^2\left[-\frac{5}{3}+(1-\mu)^2\right] & -a(1-\mu)^2 & a^2\left[\frac{5}{3}+(1-\mu)^2\right] \end{bmatrix} \quad (2)$$

$$\mathbf{K}_s^e = \frac{ak_0}{210} \begin{bmatrix} 156-18\mu+2\mu^2 & a(44-11\mu+2\mu^2) & 54+18\mu-2\mu^2 & a(-26-11\mu+2\mu^2) \\ a(44-11\mu+2\mu^2) & a^2(14+2(1-\mu)^2) & a(26+11\mu-2\mu^2) & a^2(-14+2(1-\mu)^2) \\ 54+18\mu-2\mu^2 & a(26+11\mu-2\mu^2) & 156-18\mu+2\mu^2 & a(-44+11\mu-2\mu^2) \\ a(-26-11\mu+2\mu^2) & a^2(-14+2(1-\mu)^2) & a(-44+11\mu-2\mu^2) & a^2(14+2(1-\mu)^2) \end{bmatrix} \quad (3)$$

$$\mathbf{Q}^e = \left[ qa \quad \frac{1}{3}qa^2 \quad qa \quad -\frac{1}{3}qa^2 \right]^T \quad (4)$$

Standard finite element procedures are applied to the global analysis. In the literature of pneumatic-cable structures one can find much more complicated finite element models, usually with the use of commercial FE systems (Pedretti et al., 2004, Luchsinger and Cretol, 2006).

## 3. Parametric analysis of the pneumatic-cable bridge

The bridge studied is composed of two cylindrical pneumatic-cable beams with the radius  $R_0$ . The total cross-section, the cross-section of steel beam and the load of the Suzuki SX-4 car are presented in Fig. 2.

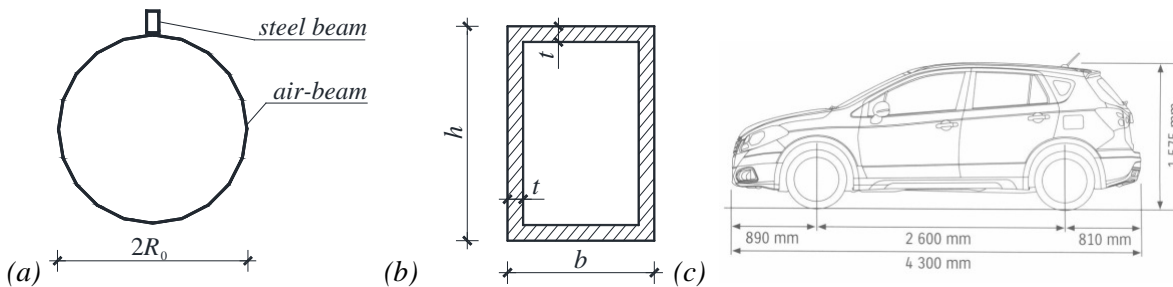


Fig. 2: Total cross-section (a), cross-section of the steel beam (b), load applied – car Suzuki SX-4 (c).

Computational model of the bridge is a simply supported beam resting on one parameter elastic foundation (Fig. 3a). The finite element model (Fig. 3b) consist of 4 elements with 10 d.o.f. Total span of the beam is  $L$ , beam weight is  $q$ , car weight is  $2P$  and  $2\alpha + 2\beta = 1$ .

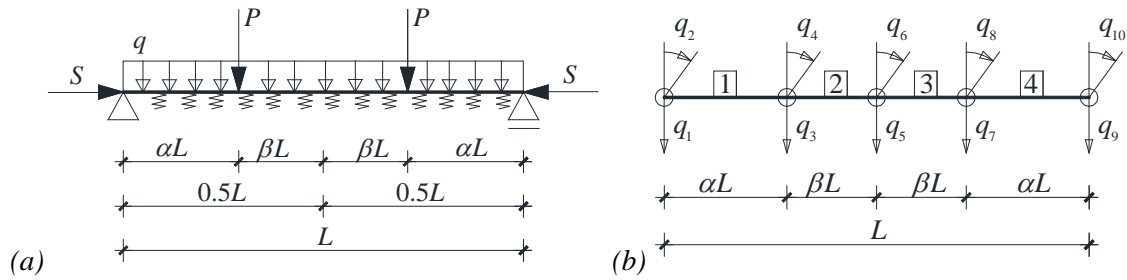


Fig. 3: Computational model (a) and finite element discretisation SX-4 (b).

The objective of the parametric analysis is to define which parameters of the pneumatic-cable beam are more or less significant for the static response of the structure. The considered parameters are: pressure of the air-beam –  $p$ , air-beam radius –  $R_0$ , height of the top steel beam –  $h$ , span of the beam –  $L$ . The following the literature of pneumatic-cable beams (Luchsinger et al., 2004 and Pedretti et al., 2004) the following relations are valid:  $k_0 = p\pi$ ,  $S = \frac{qL^2 + 2PL}{16R_0}$ . The constant data are: width of the steel beam

$b = 75$  mm (Fig. 2a), thickness of the thin-walled cross section  $t = 2.3$  mm (Fig. 2a), Young’s modulus of the steel beam  $E = 210$  GPa, Poisson’s ratio of the steel beam  $\nu = 0.3$ , mass density of steel  $7.85$  kN/m<sup>3</sup>. Reference values are: span of the beam  $L = 10$  m, air-beam radius  $R_0 = 50$  cm. The values of other parameters are shown in figures. Vertical displacement in the middle of the span is analysed. As it was proofed by Pedretti (2004) there is no danger to reach critical value of the force  $S$  because of elastic support of the steel beam on the air-beam.

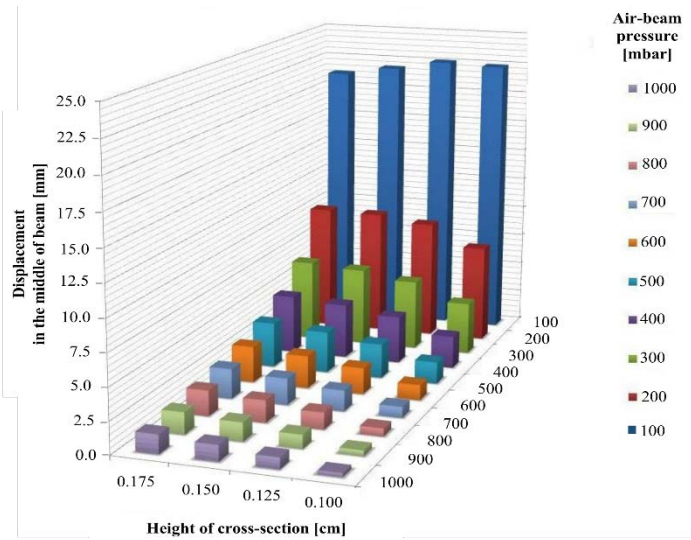


Fig. 4: Vertical displacement as a function of the height of cross-section and air-beam pressure.

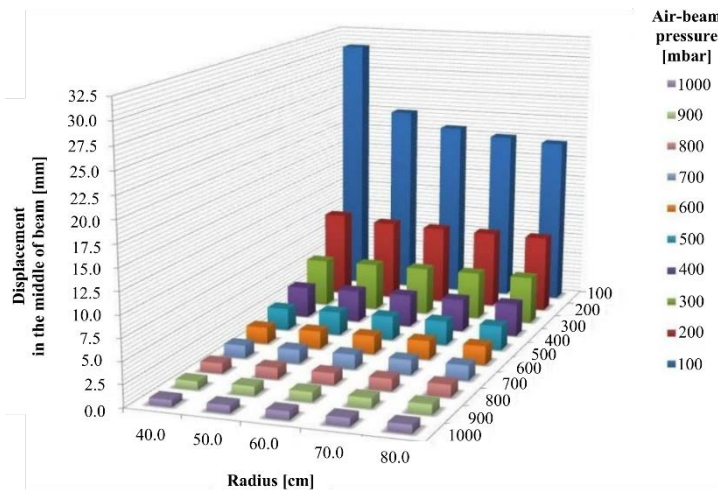


Fig. 5: Vertical displacement as a function of the radius and air-beam pressure.

Vertical displacement in the middle of beam strongly depends on the pressure applied and less on the height of the steel beam (Fig. 4). Flexible steel beam gives smaller displacements because of very stiff air-beam, with the opposite tendency for very low pressure of 100 mbar.

Maximum displacement strongly depends on the radius of the air-beam only for small pressure (Fig. 5). A similar tendency is observed if the span of the beam is concerned (Fig. 6).

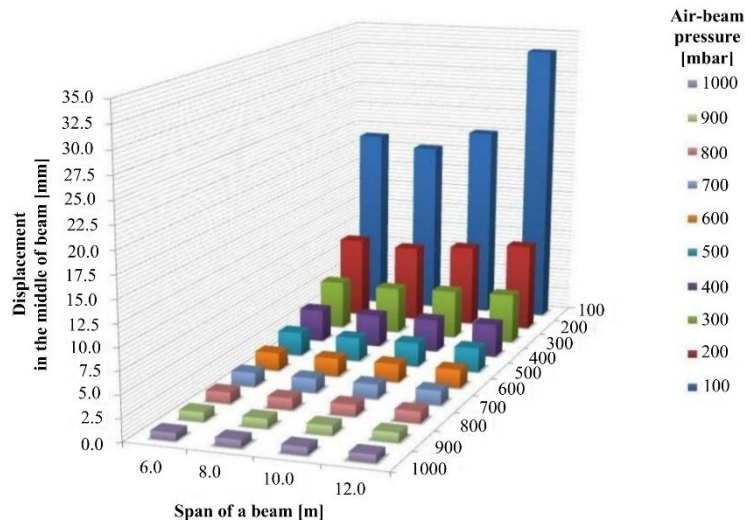


Fig. 6: Vertical displacement as a function of the span of a beam and air-beam pressure.

#### 4. Conclusions

The analysis presented in the paper is provided with the use of a simple and efficient beam finite element model. Displacements of the pneumatic-cable beams are very small for reasonable air pressure. The studied beam example gives qualitative as well as quantitative information how important can be the selected parameters for the static response of the structure.

*Pressure* is the key parameter. Displacement rapidly decrease in the range over 300 mbar.

*Height of cross-section of steel beam* is an important parameter – flexible beams are recommended.

*Span of the steel beam and radius of the air-beam* are the parameters of less importance.

As it is described in the literature of the subject (Luchsinger and Cretol, 2006, De Laet et al., 2008, Biernacka and Gilewski, 2018) the crucial aspects of the pneumatic-cable beams in real-life applications are: materials, details of construction with special emphasis of connections as well as the evolution of the shape of air-beam, which can be a subject of future computational considerations.

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