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NUMERICAL SIMULATION OF THE GAS FLOW THROUGH THE FENCES AND POROUS MEDIA

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Abstract: *The paper is focused on the numerical simulation of the compressible gas flow through the fences and the porous media. We assume the non-stationary viscous compressible fluid flow, described by the RANS equations. In order to simulate the flow through the porous media we modify the source term, achieving the characteristic loss of momentum. For the simulation of the thin fence it is possible to use the modification of the face flux. The original approach was presented recently by the authors, analysing the modification of the Riemann problem with one-side initial condition, complemented with the Darcy's law and added inertial loss. Here we are also interested in the estimate of forces acting on the diffusible barrier (fence) with given parameters. The presented examples were obtained with the own-developed code for the solution of the compressible gas flow.*

Keywords: compressible gas flow, the RANS equations, porous media, fences.

1. Introduction

The physical theory of the compressible fluid motion is based on the principles of conservation laws of mass, momentum, and energy. The mathematical equations describing these fundamental conservation laws form a system of partial differential equations (the Euler equations, the Navier-Stokes equations, the Navier Stokes equations with turbulent models). We choose the well-known finite volume method to discretize the analytical problem, represented by the system of the equations in generalized (integral) form. To apply this method we split the area of the interest into the elements, and we construct a piecewise constant solution in time. The crucial problem of this method lies in the evaluation of the so-called fluxes through the edges/faces of the particular elements. In order to discretize such fluxes for the simulation of the diffusible barrier we have shown the modification of the Riemann problem, see Kyncl and Pelant (2013, 2017). Here we present other simple method for the construction of the flux through such faces, shown also in Kyncl and Pelant (2018). This method was implemented into own computational code, and used in the numerical examples.

2. Formulation of the Equations

The system of conservation laws can be written in the following vector form

$$\frac{\partial \boldsymbol{w}}{\partial t} + \sum_{s=1}^{3} \frac{\partial \boldsymbol{f}_s(\boldsymbol{w})}{\partial x_s} = \sum_{s=1}^{3} \frac{\partial I\!R_s(\boldsymbol{w}, \nabla \boldsymbol{w})}{\partial x_s} + \boldsymbol{S}(\boldsymbol{w}) \quad \text{in } Q_T = \Omega \times (0, T). \tag{1}$$

Here $\boldsymbol{w} = \boldsymbol{w}(x,t)$ is the state vector, $x \in \Omega$, t denotes the time, Q_T is the space-time cylinder, $\boldsymbol{f}_s$ are the inviscid fluxes, $I\!R_s$ are the viscous fluxes, $\boldsymbol{S}$ is the source-term vector. Further we use the equation of state of ideal gas, and the turbulent model equations.

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The simple porous media can be simulated via the new source term, written as

$$
\boldsymbol{S}_{PM}(\varrho, \boldsymbol{v}) = \begin{pmatrix} 0 \\ -\frac{\mu}{\alpha} v_1 - C_0 \frac{\varrho}{2} |\boldsymbol{v}| v_1 \\ -\frac{\mu}{\alpha} v_2 - C_0 \frac{\varrho}{2} |\boldsymbol{v}| v_2 \\ -\frac{\mu}{\alpha} v_3 - C_0 \frac{\varrho}{2} |\boldsymbol{v}| v_3 \\ -\frac{\mu}{\alpha} \boldsymbol{v}^2 - C_0 \frac{\varrho}{2} |\boldsymbol{v}| \boldsymbol{v}^2 \end{pmatrix} \tag{2}
$$

Here α is the permeability coefficient, C_o is the pressure gradient coefficient. For the simulation of the thin fence with small viscous effect we choose large value of the coefficient $\alpha = 1e7$, which leads to negligible viscous term $\frac{\mu}{\alpha}$. Based on the work REYNOLDS (1969) we may estimate the parameter C_0 based on the fence porosity as $0.01\ C_0 = k_r = 0.52(1-\eta^2)/\eta^2$. Here η is the porosity parameter of the barrier. For the 30% barrier we choose $\eta = 0.3$. For the barriers 15%,30%,42%,50%,70% we may estimate $C_0 = 2300, 525, 250, 150, 55$.

3. Numerical method

For the discretization of the system we proceed as described in Kyncl and Pelant (2017). We use either explicit or implicit finite volume method (FVM) to solve the systems sequentionally. The polyhedral approximation of Ω is divided into set of closed polyhedrons with mutually disjoint interiors D_i called *finite volumes*. For two neighboring elements D_i, D_j we set $\Gamma_{ij} = \partial D_i \cap \partial D_j = \Gamma_{ji}$. By $\boldsymbol{n}_{ij}$ let us denote the unit outer normal to ∂D_i on Γ_{ij}. Let us construct a partition $0 = t_0 < t_1 < \dots$ of the time interval $[0, T]$ and denote the time steps $\tau_k = t_{k+1} - t_k$. We integrate the system (1) over the set $D_i \times (t_k, t_{k+1})$, and we use the Green's theorem.

$$
\int_{D_i} (\boldsymbol{w}(x, t_{k+1}) - \boldsymbol{w}(x, t_k))\, dx + \int_{t_k}^{t_{k+1}} \sum_{\Gamma_{ij} \in \Gamma_{D_i}} \int_{\Gamma_{ij}} \sum_{s=1}^{3} (\boldsymbol{f}_s(\boldsymbol{w}) - I\!R_s(\boldsymbol{w}, \nabla \boldsymbol{w}))\, (n_{ij})_s\, dS\, dt = 0 \tag{3}
$$

We define a finite volume approximate solution of the system studied (1) as a piecewise constant vector-valued functions. By $\boldsymbol{w}_i^k$ we denote the value (constant) of the approximate solution on D_i at time t_k. Using the *Riemann problem for the split Euler equations* it is possible to approximate the state vector $\boldsymbol{w}$ at the center of the edge Γ_{ij} at the time instant t_l, denoting as $\boldsymbol{w}^l_{\Gamma_{ij}}$. The inviscid face flux can be approximated as

$$
\int_{\Gamma_{ij}} \sum_{s=1}^{3} \boldsymbol{f}_s(\boldsymbol{w}(x, t_l))(n_{ij})_s\, dS \approx |\Gamma_{ij}| \sum_{s=1}^{3} \boldsymbol{f}_s(\boldsymbol{w}^l_{\Gamma_{ij}})(n_{ij})_s. \tag{4}
$$

For the approximation of the fluxes through the boundary faces see Kyncl (2011). Now it is possible to approximate the system (3) by the following explicit finite volume scheme

$$
(\boldsymbol{w}_i^{k+1} - \boldsymbol{w}_i^k) \quad + \quad \frac{\tau_k}{|D_i|} \sum_{\Gamma_{ij} \in \Gamma_{D_i}} |\Gamma_{ij}| \left(\sum_{s=1}^{3} \boldsymbol{f}_s(\boldsymbol{w}^k_{\Gamma_{ij}})(n_{ij})_s - \sum_{s=1}^{3} I\!R_s(\boldsymbol{w}^k_{\Gamma_{ij}}, \nabla \boldsymbol{w}^k_{\Gamma_{ij}})(n_{ij})_s \right) = 0. \tag{5}
$$

The **implicit scheme** was shown in Kyncl and Pelant (2017). With this finite volume formula one computes the values of the approximate solution at the time instant t_{k+1}, using the values from the time instant t_k, and by evaluating the values $\boldsymbol{w}^k_{\Gamma_{ij}}$ at the faces Γ_{ij}. In order to achieve the stability of the used method, the time step τ_k must be restricted by the so-called CFL condition, see M. Feistauer (J. Felcman). The crucial problem of this discretization lies with the evaluation of the edge values $\boldsymbol{w}^k_{\Gamma_{ij}}$ and faces fluxes.

3.1. Flux through the fence, discretization

Simpliest way to include the porous media within the FVM is to choose the elements (porous area), where the source term (2) is applied. At the face Γ_{ij} simulating the diffusible barrier (fence) we may use the modification of the face flux (4) as

$$
\int_{\Gamma_{ij}} \sum_{s=1}^{3} \boldsymbol{f}_s(\boldsymbol{w}(x, t_l))(n_{ij})_s\, dS \approx |\Gamma_{ij}| \sum_{s=1}^{3} \boldsymbol{f}_s(\boldsymbol{w}^l_{\Gamma_{ij}})(n_{ij})_s + |\Gamma_{ij}| \frac{d}{2} \boldsymbol{S}_{PM}|_{\Gamma_{ij}}. \tag{6}
$$

Here $|\Gamma_{ij}|\frac{d}{2}$ is the volume of the non-existing (artifitial) element adjacent to the face, representing the diffusible barrier (fence) with diameter d, and $\boldsymbol{S}_{PM}|_{\Gamma_{ij}} = \boldsymbol{S}_{PM}(\varrho_{\Gamma_{ij}}, \boldsymbol{v}_{\Gamma_{ij}})$, where $\varrho_{\Gamma_{ij}}$, $\boldsymbol{v}_{\Gamma_{ij}}$ are the approximated values of density and velocity vector at the face at time instant t_l.

4. Estimation of the force acting on the barrier

In our recent research we studied the damping effect of the barrier. The numerical and wind tunel experiments were made to simulate the flow behind the barrier near the ground with logarithmic distribution of incident velocity. The aim was to slow down the wind speed using the fences, and to estimate the acting forces. Here we present the acting force estimate based on the following simple example. Let us suppose the gas flow with the total temperature $\theta_0 = 293.15$K, total pressure $p_0 = 101325$ Pa, and given velocity magnitude u_∞ (freestream values). Let us assume the 50% fence composed of the set of verically placed rigid plates, totalling the height of 10cm. Our aim is to compute/ estimate the force acting on such fence. The simulations were made for the fence composed of 9,26,51, and 101 plates, denoting as D9,D26,D51, and D101. The $k - \omega$ (Kok) turbulence model was used. The flow behind such barriers is highly non-stationary, and the resulting acting force is varying. This variance is shown in figure 1. Based on the these

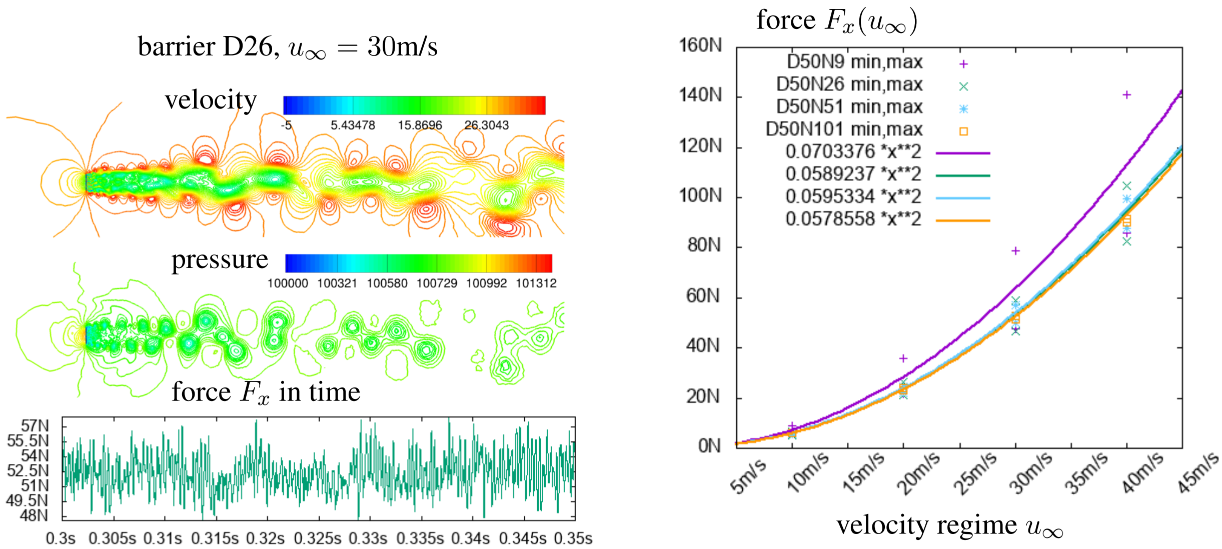

Fig. 1: Numerical simulations of turbulent flow, k-ω (Kok) turbulence model. Left: velocity and pressure field behind the D26 barrier, $u_\infty = 30$m/s, force acting on the barrier in time. Right: variation of the force F_x acting on the barrier depending on the regime velocity u_∞, barriers D9, D26, D51, and D101, h=10cm, the average force $F_x(u_\infty)$ values were fitted with the quadratic curves.

CFD simulations, it is possible to use rough approximation of the force acting on the barrier as

$$F_x = \int_S K u_\infty^2 \, dS. \tag{7}$$

To be more precise, the fluid density should be also included, as shown in Kyncl and Pelant (2018). The estimated value of the coefficient K for the considered 50% barriers lyes in the range of 0.57-0.71.

Further we simulate the flow through the fences with the use of porous media sources, setting the parameter $C_0 = 2300, 525, 250, 150, 55$. The resulting force is then computed by integration of the source term $\boldsymbol{S}_{PM}$ at the barrier. The Figure 2 shows that the computed force can be also approximated by quadratic curve (7). The estimated values for the force coefficients K of the considered permeable barriers are shown in table 1. This approach has lower computational requirements than the simulation of the fence composed of particular plates, and yields almost stationary field behind the barrier.

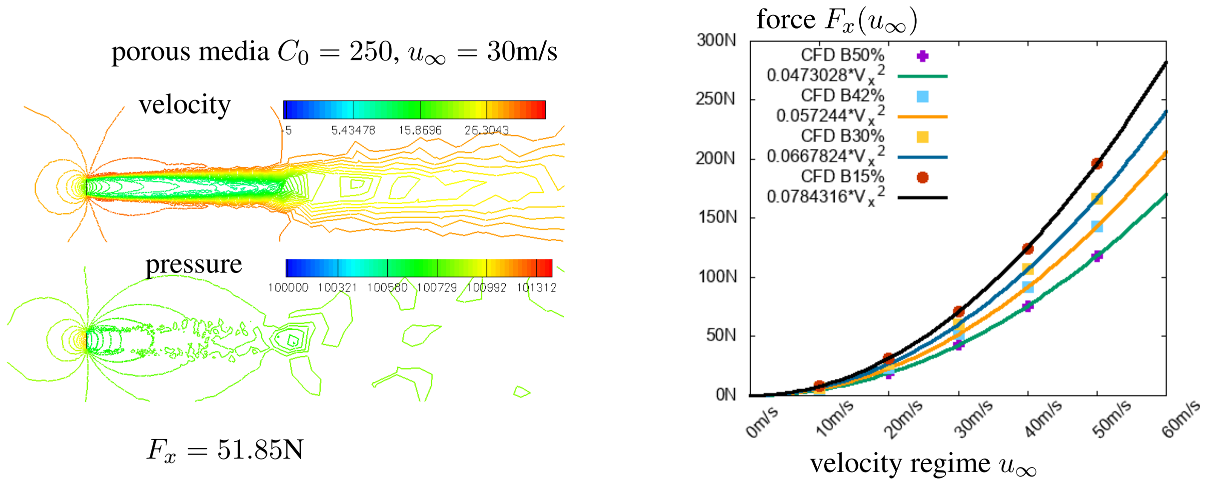

Fig. 2: Porous media simulation. Left: velocity and pressure field behind the $C_0 = 250$ barrier, $u_\infty = 30m/s$ Right: force F_x acting on the barrier depending on the regime velocity u_∞, barriers 50%,42%,30%,15% (from top left), the computed values are fitted with the quadratic curves.

BARRIER	15%	30%	42%	50%	C200A5e-9	C100A1e-8	C50A5e-8
C_0	2300	525	250	150	200	100	50
K	0.784316	0.667824	0.57244	0.473028	0.646404	0.51678	0.284178

Tab. 1: Coefficients for the quadratic estimation of the force (7) acting on the barrier (fence) with given parameters.

5. Conclusion

This paper is focused on the numerical simulation of the viscous compressible gas flow. The FVM discretization of the equations describing the flow through the porous media was shown, together with the simple estimate of the force acting on the fence. All codes were implemented into own-developed software. The presented examples show that the used simplifications cannot fully describe the complexity of the real non-stationary flow through the porous media, yet it may be used to approximate the dumping effect of such barrier.

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