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A STOCHASTIC SOLUTION TO THE DYNAMICS OF LOAD MOVEMENT ON A TRANSPORT LINE

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Abstract: *The aim of this paper is a stochastic dynamic solution for the movement of a load on a transport line. The movement of a load begins on an inclined plane, then it proceeds on a horizontal plane and ends with its stop (bump) into a spring. Basic inputs and outputs are given by statistical histograms, which describe the real variability of the variables with sufficient accuracy. The obtained results (e.g. velocity, acceleration, impact forces, etc.) provide real information needed to evaluate the design options of a transport line. The practical solution is done using the Monte Carlo Method. This work is of importance in the "Karakuri" solution of technical design, or eventually for transport line solutions for a company's production.*

Keywords: Dynamics, Stochastics model, Probability, Monte Carlo Method, Transport line, Design.

1. Introduction

Transport lines based on the "Karakuri" or gravitational principle have great advantages in terms of efficiency, as they do not need electric power, complex automatization or human resources. Ideally, the transport line should operate completely without any external interferences. Using a stochastic (probabilistic) approach, the real application of the transport line can be considered, according to the actual possibilities of the operation, see Fig.1. For example, load weight, initial conditions, friction ratios, etc. may vary over time (whether scheduled or unplanned) during everyday operation. For this reason, it is advisable to apply a probabilistic method in the design, e.g. the Monte Carlo method, and try to describe and evaluate all possible states of a line.

In our case, we are solving the final stage of manufactured product (load) transport, where these loads move along the transport line and hit against an elastic spring element, whose function is to minimize the impact. The aim is to compile and solve the kinetic equations of the dynamics using the probabilistic Monte Carlo Method, where the variability of results obtained gives the designers an appropriate description of the real situation for the design and helps them to evaluate the possibilities for optimizing its operation.

Fig. 1: Transport line in a production company

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2. Input Variables

As input values, the following variables are selected, see Fig.2 and Tab.1.

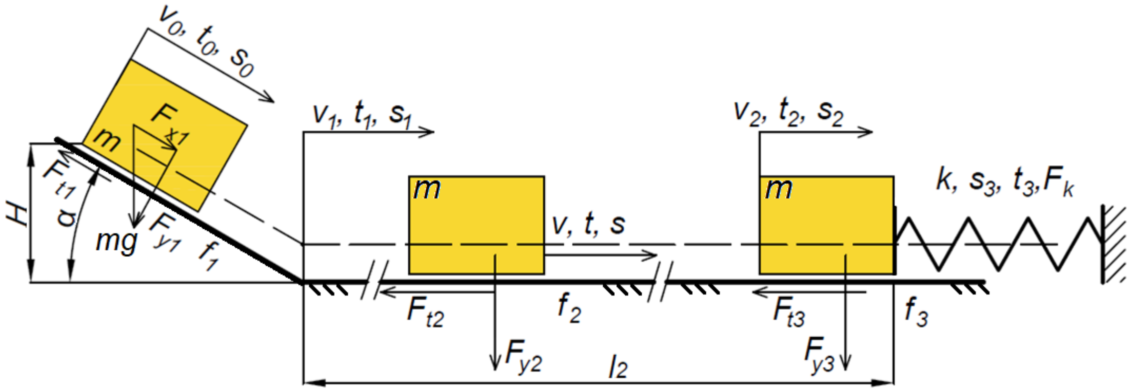


Fig. 2: Model diagram illustrating moving of a load

Tab. 1: Input Variables

Quantity	**Value**	**Distribution functions:**	**Histogram**
Load weight m [kg]	100 ± 30	Normal distribution	
Plane inclination α [°]	30 ± 3	Normal distribution	
Starting position of load H [m]	1 ± 0.1	Normal distribution	
The initial velocity of a load v_0 [ms^{-1}]	$0.1^{+0}_{-0.1}$	Uniform distribution	
Initial load path s_0 [m]	$0.1^{+0}_{-0.1}$	Uniform distribution	
Inclined plane friction coefficient (section 1) f_1 [1]	0.2 ± 0.02	Normal distribution	
Horizontal plane friction coefficient (section 2) f_2 [1]	0.1 ± 0.01	Normal distribution	
Horizontal plane friction coefficient (section 3) f_3 [1]	0.3 ± 0.03	Normal distribution	
Horizontal trajectory to impact l_2 [m]	4 ± 0.4	Normal distribution	
Spring constant k [Nm]	1000 ± 200	Normal distribution	
Gravity acceleration g [ms^{-2}]	$9.807^{+0.025}_{-0.027}$	Uniform distribution	
Start time t_0 [s]	0		

3. Dynamic Model Derivation

The analytical model must be divided into three sections, see Fig.2 and Tab.2, time $t \in < t_0, t_1 >$, $< t_1, t_2 >$ and $< t_2, t_3 >$ [s] (i.e. the solution of the three differential equations of the dynamics are derived in accordance with the normal procedures used for solid mechanics); the first section is the movement of load along the inclined plane, the second movement along the horizontal plane and the last section is the movement along the horizontal plane with a subsequent impact on the spring, where the solution of the line ends by stopping the whole body. The result of this paper is a stochastic evaluation of the trajectory motion, speed, acceleration, the time of the load motion, force ratios, impact and spring compression.

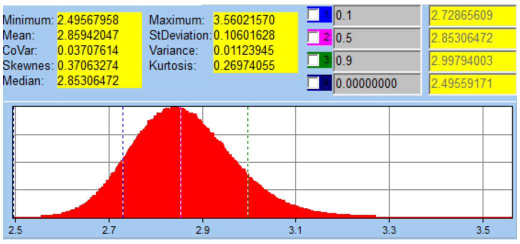
Tab. 2: Derived solution

Forces:	$F_{x1} = mg\sin(\alpha), F_{t1} = f_1\, mg\cos(\alpha), Ft_{2,3} = mgf_{2,3}$
First section: Uniformly accelerated motion	$t \in < t_0, t_1 >, s_1 = \frac{H}{\cos(\alpha)} + s_0$ see Fig.2
Equation of acceleration:	$a = \frac{d^2 s}{dt^2} = \frac{dv}{dt} = v\frac{dv}{ds} = \frac{F_{x1} - F_{t1}}{m}$
Velocity equation:	$v = \frac{ds}{dt} = \frac{F_{x1} - F_{t1}}{m} t + v_0$
Trajectory equation:	$s = \frac{F_{x1} - F_{t1}}{m}\frac{t^2}{2} + v_0 t + s_0$
Velocity equation at the end of the first section:	$v_1 = \sqrt{2\left(\frac{F_{x1} - F_{t1}}{m}(s_1 - s_0) + \frac{v_0^2}{2}\right)}$
Time at the end of the first sectiont_1.	$t_1 = \frac{(v_1 - v_0)m}{F_{x1} - F_{t1}}$
Second section: Uniformly slowed motion	$t \in < t_1,\ t_2 >, s_2 = s_1 + l_2$; see Fig.2
Equation of acceleration:	$a = \frac{d^2 s}{dt^2} = \frac{dv}{dt} = v\frac{dv}{ds} = -\frac{F_{t2}}{m}$
Velocity equation:	$v = \frac{ds}{dt} = v_1 + \frac{F_{t2}}{m}(t_1 - t)$
Trajectory equation:	$s = \left(v_1 + \frac{F_{t2}}{m} t_1\right) t - \frac{F_{t2}}{m}(t_1^2 + t^2) + s_1 - v_1 t_1$
Velocity at the end of the second section v_2:	$v_2 = \sqrt{2\left(\frac{v_1^2}{2} + \frac{F_{t2}}{m}(s_2 - s_1)\right)}$
Time at the end of the second section t_2:	$t_2 = \frac{(v_2 - v_1)m}{F_{t2}} + t_1$
Third section:	$t \in < t_2, t_3 >, v_3 = 0$; viz Fig.2
Equation of acceleration:	$a = \frac{-F_{t3} - k(s - s_2)}{m};$
Velocity equation:	$v = v_2\, cos\big(\Omega_o(t - t_2)\big) - \frac{F_{t3}}{k}\Omega_0\, sin(\Omega_0(t - t_2));\ \Omega_o = \sqrt{\frac{k}{m}}$
Trajectory equation:	$s = \frac{v_2}{\Omega_0} sin(\Omega_o(t - t_2)) + \frac{F_{t3}}{k} cos(\Omega(t - t_2)) - \frac{F_{t3}}{k} + s_2$
Time at the end of the third section t_3:	$t_3 = \frac{\operatorname{atan}\left(\frac{v_2 m}{\Omega_0 F_{t3}}\right)}{\Omega_0} + t_2$
Total trajectory of the load s_3:	$s_3 = \frac{v_2}{\Omega_0} sin(\Omega_o(t_3 - t_2)) + \frac{F_{t3}}{k} cos(\Omega(t_3 - t_2)) - \frac{F_{t3}}{k} + s_2$

Where: t= time [s], $v_{1,2,3}$= Velocity at the end of the integration step [ms^{-1}], $s_{1,2,3}$= Trajectorys at the end of the integration step [m], $t_{1,2,3}$= Times at the end of the integration step [s].

4. Application of the Probabilistic Method to Dynamic Model

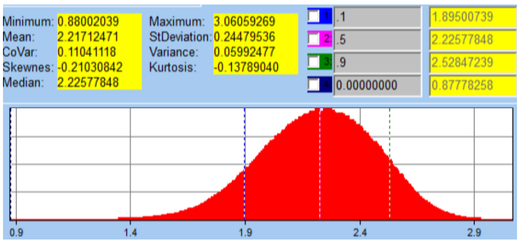
The Monte Carlo method was used to solve all three sections (Anthill sw) for 10^6 random simulations according to the relations in Tab.2, see Fig 3 to 6. The essential results obtained are the values of the instant velocity in the moment of the impact of the load on the spring, the spring compression and the force which the spring develops, and also the total time of the movement.



Minimum: 2.49567958
Mean: 2.85942047
CoVar: 0.03707614
Skewnes: 0.37063274
Median: 2.85306472
Maximum: 3.56021570
StDeviation: 0.10601628
Variance: 0.01123945
Kurtosis: 0.26974055

☐ 1: 0.1 2.72865609
☐ 2: 0.5 2.85306472
☐ 3: 0.9 2.99794003
☐ 4: 0.00000000 2.49559171

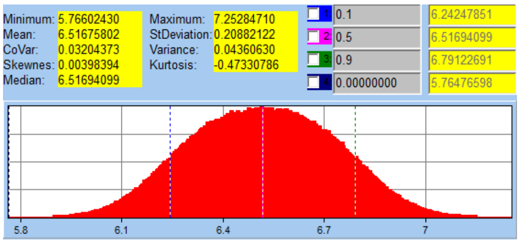
Fig.3 Motion Time (stopping the load)
$t_3 = 2.853^{+0.707}_{-0.357}\ s$



Minimum: 0.88002039
Mean: 2.21712471
CoVar: 0.11041118
Skewnes: -0.21030842
Median: 2.22577848
Maximum: 3.06059269
StDeviation: 0.24479536
Variance: 0.05992477
Kurtosis: -0.13789040

☐ 1: .1 1.89500739
☐ 2: .5 2.22577848
☐ 3: .9 2.52847239
☐ 4: 0.00000000 0.87778258

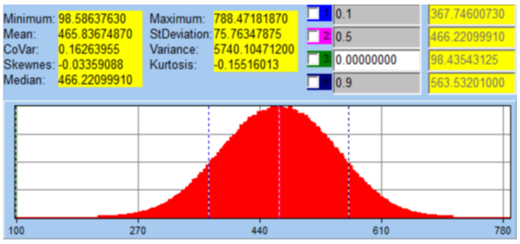
Fig.4 Spring Impact Velocity
$v_2 = 2.226^{+0.835}_{-1.346}\ ms^{-1}$



Minimum: 5.76602430
Mean: 6.51675802
CoVar: 0.03204373
Skewnes: 0.00398394
Median: 6.51694099
Maximum: 7.25284710
StDeviation: 0.20882122
Variance: 0.04360630
Kurtosis: -0.47330786

☐ 1: 0.1 6.24247851
☐ 2: 0.5 6.51694099
☐ 3: 0.9 6.79122691
☐ 4: 0.00000000 5.76476598

Fig.5 Total trajectory of the load (in time t_3)
$s_3 = 6.517^{+0.736}_{-0.751}$ m



Minimum: 98.58637630
Mean: 465.83674870
CoVar: 0.16263955
Skewnes: -0.03359088
Median: 466.22099910
Maximum: 788.47181870
StDeviation: 75.76347875
Variance: 5740.10471200
Kurtosis: -0.15516013

☐ 1: 0.1 367.74600730
☐ 2: 0.5 466.22099910
☐ 3: 0.00000000 98.43543125
☐ 4: 0.9 563.53201000

Fig.6 Directional force of the spring
$F_k = k(s_3 - s_2) = 466.2^{+332.3}_{-367.6}$ N

For example, the probabilistic reliability assessment can be carried out by means of the reliability function $RF = F_{allowable} - F_k$, where $F_{allowable}$ [N] is the maximum allowable force in spring. Hence, the probability of failure is the probability that F_k will exceed $F_{allowable}$, i.e. $P_f = P(RF \leq 0)$.

5. Conclusion

Using the stochastic approach, it was possible to model a dynamic system for the load travel on the transport line, including stopping the load with an impact. The results obtained give a real idea of the interaction properties for the transport line and the load, and these are a valuable source of information when designing operation, redeveloping or optimizing a production process. This means that e.g. during operation, you can expect a traffic time between $2.853^{+0.707}_{-0.357}$ s or impact force F_k in the range $466.2^{+332.3}_{-367.6}$ N. Stochastic mechanics is not a very common method used in engineering designs. For this reason, the presented procedures represent a benefit, as they are in line with the latest trends in science and research. Further experience with stochastic modelling can be found in e.g. Cienciala 2017 and Frydrýšek 2017.

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