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PHASE-FIELD DAMAGE OF LAMINATED GLASS PLATES

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Abstract: *Laminated glass is used in automotive industry for almost a centrury. Recently its use has expanded widely in civil engineering, where it play structure role. For safe and efficient design of laminated glass elements it is important to implement material models which correctly predict its behavior including after rupture. The contribution provides theoretical background and implementation aspects of such a model. It is focused mainly on post-breakable behavior and paves the way for extension to impact loading.*

Keywords: Phase-field damage, laminated glass, Mindlin plate, brittle fracture

1. Introduction

Laminated glass consist of several glass plies and polymer interlayers. The glass behaves almost perfectly elastically until it breaks. The laminated setup improves load-bearing capacity after rupture and prevents collapse. This fact calls for a material model which is capable of predicts and modeling crack initialization and propagation. Such a model is presented in the contribution. Since the shear is dominant stress state in the interlayer we employ the Mindlin-Reissner plate model. The phase-field crack model is used for damage prediction. Two phase-field models are used, proposed by Bourdin (2008) and Pham (2011). Finally, in coupling damage and the plate theory we follow footsteps of Kiendl (2016), moreover dynamic extension is based on Hofacker (2012).

2. Laminated glass plates

The individual glass layer is modeled as the Mindlin-Reissner plate. Its mathematical formulation is well-known, but we recall basic equations for the following extension to damage. We assume that the mid-surface lies in the $x-y$ plane and is characterized by vector set $(x, y)^{\mathrm{T}} = \mathbf{x} \in \Omega = \langle 0, L_1\rangle \times \langle 0, L_2\rangle$. The thickness of the plate in z direction is constant equal to h. For compact notation we use bold symbols $\mathbf{u} = (u, v)^{\mathrm{T}}$ and $\boldsymbol{\varphi} = (\varphi_x, \varphi_y)^{\mathrm{T}}$ for in plane displacements and rotations. Basic kinematic description of the Mindlin-Reissner plate is then given by

$$\mathbf{u}(\mathbf{x}, z) = \mathbf{u}_0(\mathbf{x}) + \mathbf{S}\boldsymbol{\varphi}(\mathbf{x})z, \quad w(\mathbf{x}, z) = w_0(\mathbf{x}), \tag{1}$$

where $\mathbf{u}_0 = (u_0, v_0)^{\mathrm{T}}$ is in plane membrane displacement field and $\mathbf{S}$ is antisymmetric matrix

$$\mathbf{S} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \tag{2}$$

The strains are given in classic manner as

$$\boldsymbol{\varepsilon}(\mathbf{x}, z) = \nabla_{\mathrm{s}}\mathbf{u}(\mathbf{x}, z) = \nabla_{\mathrm{s}}\mathbf{u}_0(\mathbf{x}) + \nabla_{\mathrm{s}}\mathbf{S}\boldsymbol{\varphi}(\mathbf{x})z, \quad \boldsymbol{\gamma}(\mathbf{x}) = \frac{1}{2}\left(\nabla w_0(\mathbf{x}) + \mathbf{S}\boldsymbol{\varphi}(\mathbf{x})\right), \tag{3}$$

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where ∇_s represents symmetric gradient operator. Constitutive equations for stresses are based on the plane stress assumption

$$\boldsymbol{\sigma}(\boldsymbol{\varepsilon}) = \frac{E}{1-\nu^2}\left(\nu \mathrm{tr}(\boldsymbol{\varepsilon})\mathbf{I} + (1-\nu)\boldsymbol{\varepsilon}\right), \quad \boldsymbol{\tau}(\boldsymbol{\gamma}) = 2G\boldsymbol{\gamma} \tag{4}$$

where $\mathbf{I}$ is the second-order identity tensor and E, G, ν are material constants. The solution of Mindlin/Reissner plate can be formulated as minimization of the energy functional

$$\int_V \Psi(\boldsymbol{\varepsilon}, \boldsymbol{\gamma}) = \frac{1}{2}\int_V \boldsymbol{\varepsilon} : \boldsymbol{\sigma}(\boldsymbol{\varepsilon}) + \boldsymbol{\gamma} \cdot \boldsymbol{\tau}(\boldsymbol{\gamma})\, \mathrm{d}V, \tag{5}$$

where the domain $V = \Omega \times \langle -h/2, h/2 \rangle$ corresponds the volume of the plate. Assuming constant distribution of stiffness and integrating over thickness we can rewrite (5) in traditional form of integral over Ω with following multipliers for individual members

$$\int_{-h/2}^{h/2} \mathrm{d}z = h, \quad \int_{-h/2}^{h/2} z\, \mathrm{d}z = 0, \quad \int_{-h/2}^{h/2} z^2\, \mathrm{d}z = \frac{h^3}{12}. \tag{6}$$

The laminated glass consists of n Mindlin-Reissner plates. We indicate individual layers by integer index $i \in \langle 0, n \rangle$. This layers are not independent, but are kinematically bounded. One way how to enforce kinematic restrictions is to use the method of lagrange multipliers. This is powerful concept but it introduces additional degrees of freedom. For simplicity, we bind the degrees of freedom of neighbouring plates directly by relations

$$\mathbf{u}^{(i)}(\mathbf{x}, h/2) - \mathbf{u}^{(i+1)}(\mathbf{x}, -h/2) = \mathbf{0}, \tag{7}$$

$$w^{(i)}(\mathbf{x}) - w^{(i+1)}(\mathbf{x}) = 0. \tag{8}$$

This approach reduces the number of independent degrees of freedom number. On the other hand, such formulation is unable to describe the delamination effect, more information in Zemanová (2014).

3. Phase-field damage

Due to low modulus of elasticity, flexibility and viscosity of the interlayer only glass plies appears damaged in post-breaking patterns. Therefore, we assume that the interlayer does not break. The damage of the glass layers is modeled as described below.

Phase field modelling of fracture is nowadays very popular approach. It is based on regularization of Francfort (1998) fracture functional, where sharp crack is replaced by smooth damage field ω. Displacements and damage solutions are obtain through minimization of energy functional

$$\mathcal{E}(\boldsymbol{\varepsilon}, \omega) = \int_\Omega \Psi(\boldsymbol{\varepsilon}, \boldsymbol{\gamma}, \omega)\mathrm{d}\Omega + \frac{G_\mathrm{f}}{c_0}\int_\Omega \frac{1}{l}\alpha(\omega) + l|\nabla\omega|^2 \mathrm{d}\Omega. \tag{9}$$

Parameter l is a regularization parameter, or it can play the role of the process zone width, G_f is fracture toughness, function α characterizes type of the phase-field model and c_0 is the scaling parameter. We use two type of α function

$$\alpha(\omega) = \omega, c_0 = 8/3 \quad \text{and} \quad \alpha(\omega) = \omega^2, c_0 = 2. \tag{10}$$

The former one behaves linearly until rupture, which is desirable but it leads to computationally more expensive constrained minimization. The latter case leads to linear problem without inequality constraints but it behaves non-linearly from the beginning of loading.

We assume that only tension part of elastic energy is degraded by damage field. The internal energy therefore is written as

$$\Psi(\boldsymbol{\varepsilon}, \boldsymbol{\gamma}, \omega) = g(\omega)\Psi^+(\boldsymbol{\varepsilon}, \boldsymbol{\gamma}) + \Psi^-(\boldsymbol{\varepsilon}, \boldsymbol{\gamma}), \tag{11}$$

where $g(\omega)$ is degradation function and superscripts $^+$ and $^-$ stand for tension and compression part of elastic energy density. We utilize the quadratic degradation function

$$g(\omega) = (1-\omega)^2. \tag{12}$$

Minimization of (9) leads to weak form

$$\int_V 2\,(1-\omega)\,\Psi^+\delta\omega\;\mathrm{d}V + \frac{G_\mathrm{f}}{c_0}\int_V \frac{1}{l}\alpha'(\omega)\delta\omega + 2l\nabla\omega\cdot\nabla\delta\omega\;\mathrm{d}V, \tag{13}$$

where Greek letter δ stands for variation.

4. Thickness integration

The formulations presented in sections 2 and 3 holds generally, i.e. in 3D space. Reduction to plate is performed by thickness integration on intervals $\langle -h/2, h/2\rangle$. But we assume that only tension is degraded by function $g(\omega)$ whereas compression stiffness is intact. If neutral axis is out of the cross section then stiffness is still constant across thickness. If position of neutral axis z_N lies in cross section then integration is performed on intervals $I_1 = \langle -h/2, z_\mathrm{N}\rangle$ above neutral axis and on interval $I_2 = \langle z_\mathrm{N}, h/2\rangle$ below neutral axis separately. Unfortunately position of neutral axis is different in individual in plane directions. Following Kiendl (2016) we perform numerical integration. On tension part of the cross section, reduced stiffness

$$\boldsymbol{\sigma}^+(\boldsymbol{\varepsilon}) = g(\omega)\frac{E}{1-\nu^2}\left(\langle\nu\mathrm{tr}(\boldsymbol{\varepsilon})\rangle^+\mathbf{I} + (1-\nu)\boldsymbol{\varepsilon}^+\right) \tag{14}$$

is integrated whereas on compression part it is integrated stiffness

$$\boldsymbol{\sigma}^-(\boldsymbol{\varepsilon}) = \frac{E}{1-\nu^2}\left(\langle\nu\mathrm{tr}(\boldsymbol{\varepsilon})\rangle^-\mathbf{I} + (1-\nu)\boldsymbol{\varepsilon}^-\right). \tag{15}$$

Sharp bracket $\langle\bullet\rangle^\pm$ represent positive respective negative part of scalar whereas $\boldsymbol{\varepsilon}^\pm$ are positive and negative principal strains defined through spectral decomposition as

$$\boldsymbol{\varepsilon}^\pm = \sum_{i=1}^{2}\langle\varepsilon_i\rangle^\pm\mathbf{n}_i\otimes\mathbf{n}_i, \tag{16}$$

where ε_i and $\mathbf{n}_i$ are principal strains and principal directions.

5. Dynamics

Extending the model for dynamics problems is straightforward. We only add variation of kinetic energy to bilinear form (5). After integrating over thickness it has following form

$$h\rho\int_\Omega \ddot{w}\delta w\;\mathrm{d}\Omega + h\rho\int_\Omega \ddot{\mathbf{u}}_0\cdot\delta\mathbf{u}_0\;\mathrm{d}\Omega + \frac{h^3}{12}\rho\int_\Omega(\mathbf{S}\ddot{\boldsymbol{\varphi}})\cdot(\mathbf{S}\delta\boldsymbol{\varphi})\;\mathrm{d}\Omega, \tag{17}$$

where ρ is density and double dot stands for second derivative with respect to time. To obtain solution it is necessary to discretize the formulation in time. We adopt constant acceleration Newmark method which approximates quantity in time t_{i+1} as

$$\dot{\zeta}_{i+1} = \dot{\zeta}_i + \frac{\Delta t}{2}\left(\ddot{\zeta}_{i+1} + \ddot{\zeta}_i\right), \quad \zeta_{i+1} = \zeta_i + \Delta t\dot{\zeta}_i + \frac{\Delta t^2}{4}\left(\ddot{\zeta}_{i+1} + \ddot{\zeta}_i\right). \tag{18}$$

For our task $\zeta \in \{\mathbf{u}_0, \boldsymbol{\varphi}, w\}$. From these equations it is possible to express second derivative as follows

$$\ddot{\zeta}_{i+1} = \frac{4}{\Delta t^2}\left(\zeta_{i+1} - \tilde{\zeta}_i\right), \quad \tilde{\zeta}_i = \zeta_i + \Delta t\dot{\zeta}_i + \frac{\Delta t^2}{4}\ddot{\zeta}_i. \tag{19}$$

Finally, substituting (19) into (17) we get linear equation for unknown triple $\mathbf{u}_{0,i+1}, \boldsymbol{\varphi}_{i+1}, w_{i+1}$. Final algorithm is shown as pseudo-code in algorithm 1.

Algorithm 1: Dynamic Mindlin-Reissner plate with damage

Data:
Set tolerance ξ;
initialization: $k \leftarrow 0$;
for $t \in \{t_0, \ldots, t_N\}$ **do**
  while $\xi^{(k+1)} > \xi$ **do**
    perform thickness integration ;
    calculate $\mathbf{u}_{0,t+1}^{(k+1)}, \boldsymbol{\varphi}_{t+1}^{(k+1)}, w_{t+1}^{(k+1)}$;
    calculate $\omega_{t+1}^{(k+1)}$ from (13);
    for ζ *in* $[\mathbf{u}_{0,t+1}^{(k+1)}, \boldsymbol{\varphi}_{t+1}^{(k+1)}, w_{t+1}^{(k+1)}, \omega_{t+1}^{(k+1)}]$ **do**
      update $\xi^{(k+1)} \leftarrow \max(\xi^{(k+1)}, ||\zeta^{(k+1)} - \zeta^{(k)}||)$
    end
    update $k \leftarrow k+1$;
  end
  update $\dot{\zeta}_{t+1}$ by (18) for $\zeta \in \{\mathbf{u}_0, \boldsymbol{\varphi}, w\}$;
  update $\ddot{\zeta}_{t+1}$ by (19) for $\zeta \in \{\mathbf{u}_0, \boldsymbol{\varphi}, w\}$;
  update $\tilde{\zeta}_{t+1}$ by (19) for $\zeta \in \{\mathbf{u}_0, \boldsymbol{\varphi}, w\}$;
end

6. Conclusions

In paper we summarize phase-field damage plate modeling. We want investigate impact resistant of laminated glass and investigate its behavior during post-breaking phase. Contribution summarize theoretical aspects of required simulations. It is based mainly on Kiendl (2016), which formulated phase-field damage of plates. Paper extends formulation to Mindlin-Reissner plates and adds dynamic effects. In further investigations we will focus on different approaches to computing internal forces in cross sections that are partly in tension and partly in compression.

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