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BEHAVIOUR OF EXTENDED BACKLASH BASED DRY FRICTION MODEL IN SIMULINK IN COMPARISON WITH DAHL AND LUGRE MODELS

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Abstract: *Behaviour of analytical dry friction model based on SIMULINK Backlash block which was extended to contain velocity-dependent friction coefficient characteristic is studied. Comparison with the same model but with Coulomb friction characteristic and with Dahl and LuGre models is shown. Different simulation time-step is needed for each model to reach stability when increasing motion amplitude.*

Keywords: dry friction, Backlash model, Dahl model, LuGre model, SIMULINK

1. Introduction

Dry friction model based on *Backlash* block from SIMULINK library was created and used for simulations of three-blade bundle and bladed wheel with inter-blade dry friction couplings, see Pešek et al. (2019) and Šnábl et al. (2018). This model used serially connected friction element based on Coulomb friction law and spring, see Pešek and Půst (2014), and for further research it was extended to contain also velocity-dependent friction coefficient with certain simplification described below. Comparisons with existing LuGre model, see Canudas de Wit et al. (1995), and Dahl model, see Dahl (1975), have been made.

2. Description of the Extended Backlash Based Dry Friction Model

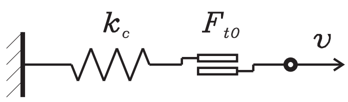

Fig. 1: Schematic picture of stick-slip friction model.

Physical representation of the dry friction model is shown on fig. 1. Friction element is serially connected to spring which allows elastic deformation for small amplitudes of vibration. *Backlash* block from SIMULINK library is used to calculate slip distance. Behaviour of this block can be described as follows:

$$\dot{y}_{sl} = \begin{cases} 0, & (y_{sl} - F_{t0}/k_c) < y_{rel} < (y_{sl} + F_{t0}/k_c) \\ \dot{y}_{rel}, & y_{rel} = y_{sl} \pm F_{t0}/k_c \end{cases}, \tag{1}$$

where relative motion of two bodies connected with dry friction coupling described by y_{rel} serves as block input, slip distance y_{sl} is block output and value $|-F_{t0}/k_c, F_{t0}/k_c| = 2F_{t0}/k_c$ serves as parameter *Deadband* of the *Backlash* block. Constant $F_{t0} = F_n\mu_0$ is Coulomb friction force with normal contact force F_n and adhesion coefficient μ_0 and k_c is stiffness of spring. Let's denote $\dot{y}_{rel} = v$ and deformation of the spring z. It is clear that:

$$z = y_{rel} - y_{sl}. \tag{2}$$

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Notice that only constant friction force F_{t0} is considered for calculating the slip distance in equation 1. A simplification has been bade here that deformation of the spring remains constant during slip phase to avoid calculating the internal contact dynamics. Then the derivation of equation 2 gives $\dot{y}_{sl} = v$ and the friction force is calculated as:

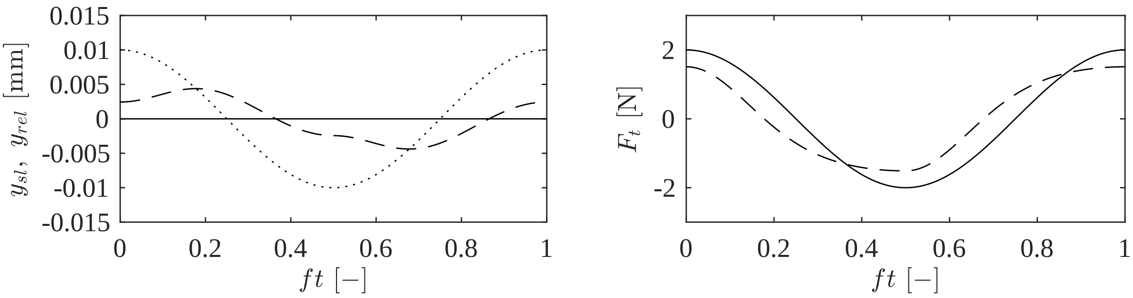
$$F_t = \begin{cases} k_c z, & \dot{y}_{sl} = 0 \\ F_n \mu(\dot{y}_{sl}), & \dot{y}_{sl} \neq 0 \end{cases}. \tag{3}$$

3. Comparison of Dry Friction Models

Tab. 1: Overview of friction contact model parameters.

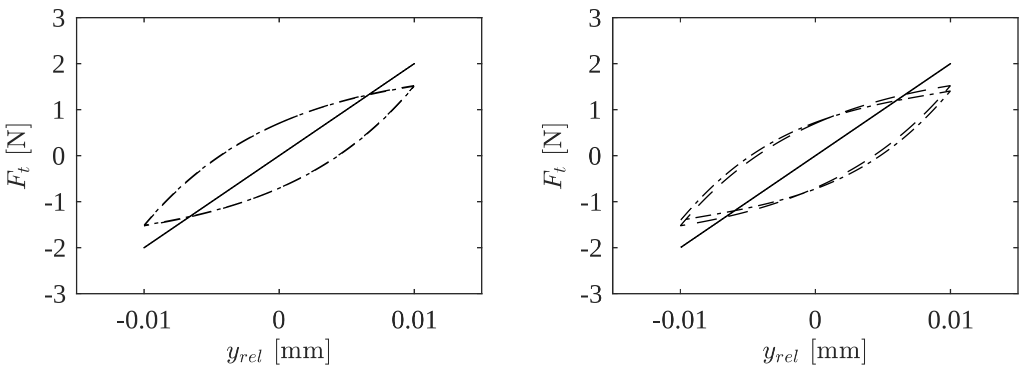
Parameter	Unit	Coulomb	Steel-brass
$k_c = \sigma_0$	$[\mathrm{Nm}^{-1}]$	$2 \cdot 10^5$	$2 \cdot 10^5$
F_{t0}	[N]	2	2
μ_0	[1]	0.2775	0.2775
$\mu(v)$	[1]	0.2775	$0.065 + 0.156 \cdot \exp^{-4.757(\lvert v \rvert - 0.065)}$

Backlash based model with Coulomb friction and steel-brass friction characteristic will be compared to Dahl and LuGre models with zero viscous damping. Parameters of the models are shown in tab. 1 where friction coefficient equation was created by exponential fitting of data stated in Sextro (2007). Response of friction element models with harmonic motion excitation $y_{rel} = a \sin(2\pi f t)$ for various amplitudes a and excitation frequencies f will be tested.



(a) Slip distance (b) Friction force

Fig. 2: Outputs of Backlash with Coulomb and Backlash with steel-brass (—) and Dahl and LuGre (−−) models for $a = 0.01$ mm and $f = 100$ Hz.

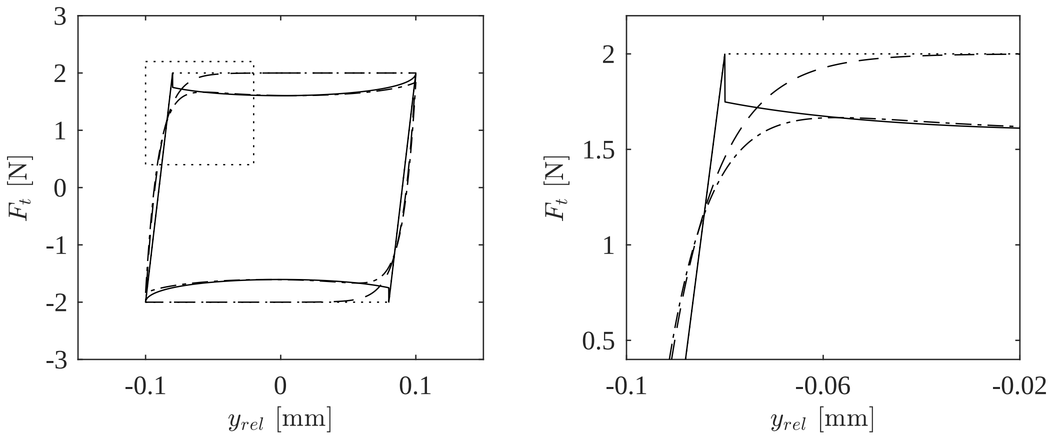


(a) $f = 100$ Hz (b) $f = 1000$ Hz

Fig. 3: Behaviour of Backlash with Coulomb (···), Dahl (−−), LuGre (−·−) and Backlash with steel-brass (—) models in pre-sliding phase, $a = 0.01$ mm.

In pre-sliding phase - for amplitudes $a \leq F_{t0}/k_c$ no slip occurs in *Backlash* based models which corresponds to the model shown on fig. 1 while in Dahl and LuGre models slip occurs which is property of the bristle models. Slip distances y_{sl} of friction models together with dotted relative displacement y_{rel} are shown on fig. 2a for amplitude $a = 0.01$ mm and frequency $f = 100$ Hz. *Backlash* based models have zero slip value while Dahl and LuGre models have the same non-zero value. Fig. 2b shows that the friction force of Dahl and LuGre models does not reach the break-away force $F_{t0} = 2$ N.

Hysteresis behaviour of the models in pre-sliding phase is visible on fig. 3 where friction force F_t is plotted as a function of relative displacement y_{rel} for all models. Notice that both *Backlash* based models have identical elastic behaviour (straight line) which is based on the definition. On the other hand non-zero slip in Dahl and LuGre models indicates energy dissipation and slip velocity causes different behaviour of these two models which is not yet evident at frequency $f = 100$ Hz on 3a because of small slip velocity but it can be clearly seen at $f = 1000$ Hz on fig. 3b.

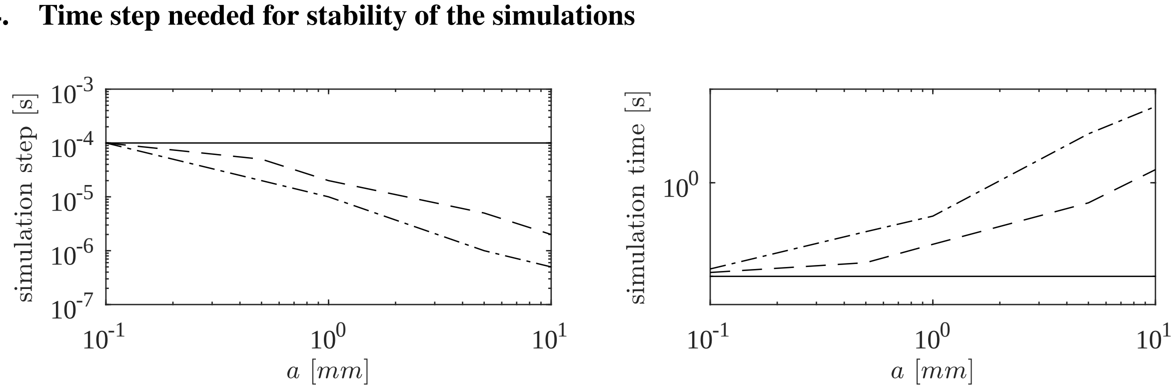


(a) Full view (b) Detail

Fig. 4: Behaviour of Backlash with Coulomb ($\cdots$), Dahl ($--$), LuGre ($-\cdot-$) and Backlash with steel-brass (—) models for $a = 0.1$ mm and $f = 100$ Hz.

Behaviour of the models during slip phase can be seen on fig. 4. *Backlash* based model with Coulomb friction has typical rhomboid hysteresis where angle of the rhomboid is given by spring stiffness k_c. Dahl model starts sliding before reaching F_{t0}, the transition between stick and slip is smooth and the friction force asymptotically approaches F_{t0} for increasing slip distance. Extended *Backlash* based model shows step decrease of friction force after transition from stick mode into slip. It is caused by neglecting of contact internal dynamics a making the approximation that $\dot{y}_{sl} \doteq v$. Similarly as Dahl model smoothly approaches *Backlash* based model with Coulomb friction, LuGre model approaches extended *Backlash* based model.

4. Time step needed for stability of the simulations



(a) Simulation step length (b) Simulation time

Fig. 5: Simulation step length and time of simulation of Backlash based models (—), Dahl ($--$) and LuGre ($-\cdot-$) models.

Dahl and LuGre models showed that simulation time step needed for stability of the simulation is amplitude-dependent.

Simulations of friction elements with sine motion excitation with frequency $f = 100$ Hz have been made for various amplitudes, longest stable time step for fixed-step ode3 solver was chosen and time of simulation was measured. Results can be seen on fig. 5.

Simulation time step 10^{-4} was chosen as maximal step for this comparison. It gives 100 samples per period. Fig. 5a shows that *Backlash* based models are stable at maximal chosen time step independently on the amplitude while both Dahl and LuGre models have their largest stable simulation step dependent on amplitude. Also the simulation time shown on fig. 5b remains constant while simulation time of Dahl and LuGre models increases rapidly for large amplitudes.

5. Conclusions

Backlash based model with Coulomb friction law has been herein compared with Dahl friction model. *Backlash* based model has sharp transition between stick and slip phase and no energy dissipates during the stick phase. Dahl model ”smoothens” the transition between stick and slip phase and in slip phase it asymptotically closes to *Backlash* based model solution. There is energy dissipation in stick phase in Dahl model.

Extended *Backlash* based model which is able to handle also velocity-dependent friction coefficient has been herein described and compared to LuGre model. Similar difference between these two models and previously mentioned models is observed. LuGre smoothens the transitions from stick to slip phase and there is energy dissipation in stick phase in Dahl model.

Biggest difference between *Backlash* based models and Dahl and LuGre models is in terms of simulation step needed for stability of the model. Fixed-step ode3 solver in SIMULINK was used to run the simulations and while the *Backlash* based models time step can be set quite high (to provide necessary resolution) independently on excitation parameters, Dahl and LuGre models suffer on unstable behaviour when increasing amplitude of vibration. Their simulation time step must be reduced for higher amplitudes and simulation time increases rapidly.

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