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TESTING OF HPD MODEL OF DISSIPATED ENERGY OF HARD RUBBERS AT TORSIONAL FINITE DEFORMATIONS AND TEMPERATURES

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Abstract: *This paper deals with the temperature dependence of hard rubber behavior on torsional harmonic loading. This behavior was described by dissipated and deformation energy of the rubber. This knowledge was used to test the validity of the hyperelastic proportional damping (HPD) in dependence on temperature and finite deformations. From the experiments, the dissipated energy was obtained from the hysteresis loop in one period of harmonic load, depending on the temperature for the given frequency. The deformation energy was determined from the shear stress analysis on the surface of the cylinder considering hyperelasticity. The constants of these models were obtained by optimizing the stress-strain curve from the shear analysis on the cylinder surface. With this optimization, the Yeoh hyperelastic model was chosen.*

Keywords: **Dissipated energy, deformation energy, hyperelasticity, temperature dependence, hysteresis loop.**

1. Introduction

Characteristic of the mechanical behaviour of rubber-like materials is commonly expressed by the constitutive equation through a strain energy density function. Many theoretical models were designed to characterize the mechanical behaviour of hyperelastic materials (Ogden, 1997). These models are divided into two categories: a) based on statistical mechanics; b) phenomenological where the material is modeled as a continuum. Tests of rubbers with higher hardness Sh 50-80 were performed in the laboratories of IT AS CR in recent years (Šulc et al., 2017). The aim of this work is to show the temperature dependence of deformation and dissipated energy of hard rubbers under torsional dynamic load with respect to large torsional deformations on the basis of a phenomenological approach. This dependence is used here to test the so-called hyperelastic proportional damping model (HPD) (Šulc et al., 2017) analogical to the model of proportional damping in the linear viscoelastic theory (Pešek et. al., 2015). The strain energy density is represented here by the polynomial Yeoh hyperelastic model. An important assumption for solving the torsional deformation energy is that the torsional is analogous to plain simple shear straining on the surface of the cylinder. The deformation energy constants of the hyperelastic model were obtained by a curve fitting of analytical to experimental stress-strain curves obtained from the shear stress analysis on the cylinder surface. This paper is focused on determination of the constant β of HPD model in dependence on temperature and finite deformations.

2. Experimental data

In the experimental part of the paper, we tested the rubber cylindrical sample of EPDM with the hardness of Sh85 under the harmonic torsional load on our designed laboratory equipment in department D3. The measurement methodology and the evaluation were presented in the paper (Šulc et al., 2016). To obtain the temperature dependence of deformation energy and the stress vs. deformations curve to optimize hyperelastic models were tested at temperatures of -50 ° C, -40 ° C, -20 ° C, 0 ° C, 20 ° C, 40 ° C, 50 ° C,

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where end temperatures are in glass (-50 ° C) and rubbery (50 ° C) region. For each temperature, the excitation frequency was changed in the range from 2 to 5 Hz. Further, for each temperature and frequency, we had the variable excitation amplitude from 0 to 15 Nm with a step of approximately 1 Nm and recorded strain from 1% to about 30%, which were the maximal strain values. From these measurements we obtained temperature dependencies of material parameters, stress vs. deformations curves and hysteresis loops for our rubber sample. From the evaluation of these experiments, temperature dependence proved to be the dominant over frequency dependence. Therefore, the excitation frequency 4 Hz for the harmonic loading of our sample was selected in this paper for presenting results. Figure 1 a) shows the experimental stress dependence on the strain drawn by the dashed line with different line thicknesses, where the thinnest line is assigned to a temperature of -20° C, the medium line to 0° C and the thickest to 20° C. Next, in Figure 1, the stress from the Yeoh hyperelastic model is obtained by fitting its constants marked by a solid line with a variable thickness as in the experiment. The effect of temperature is evident on these curves, so they show a significant deviation from linear dependence with decreasing temperature.

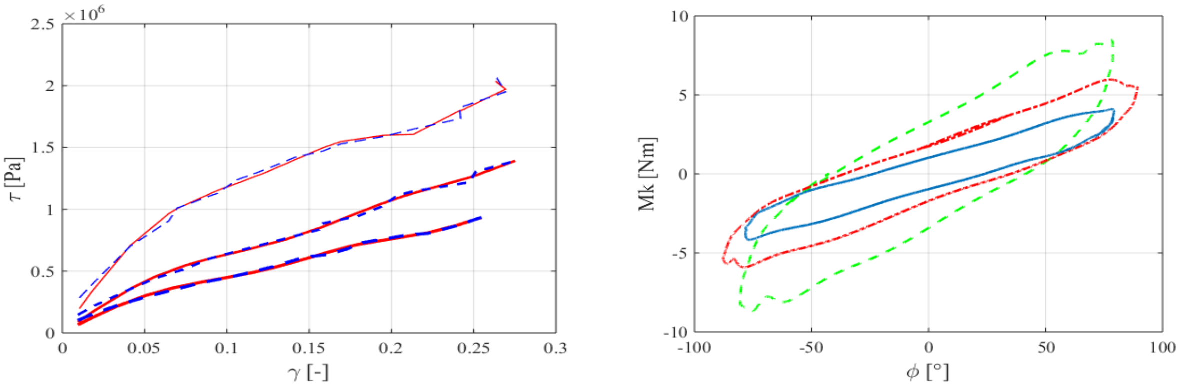

Fig. 1: a) Dependence of stress on strain from the experiment (dashed line) and stress from the Yeoh model (solid line) for temperatures of 20° C, 0° C and -20° C; b) Hysteresis loops corresponding to 21% strain for temperature 20° C (solid line), 0° C (dot-and-dash line) and -20° C (dashed line).

Figure 1 b) shows the hysteresis loops obtained for one harmonic excitation period with the same time period start (about 1s) for 20° C (solid line), 0° C (dot-and-dash line), and -20° C (dashed line). These curves correspond to a 21% strain and there is a quasi-viscous-elastic behavior of our sample. Dissipated energy was obtained from areas of hysteresis loops over the entire strain range from 1% to about 27%. The dependence of dissipated energy on strain is shown in Figure 2 for temperatures of 20° C (solid line), 0° C (dot-and-dash line) and -20° C (dashed line).

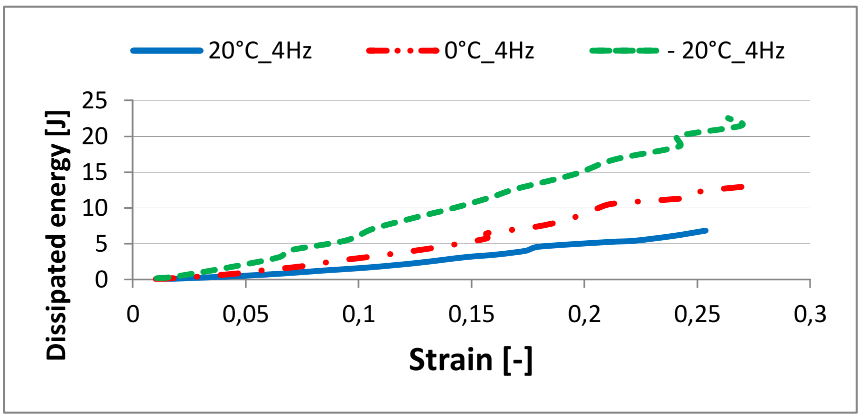

Fig. 2: Dependency of Dissipated energy vs. strain for temperature 20°C (solid line), 0°C (dot-and-dash line) and -20°C (dashed line).

3. Solution of hyperelastic proportional damping

In this part of the paper, the best hyperelastic model of deformation energy density from the numerical analysis of the shear stress on the cylinder surface for the whole range of experiments was first sought.

The assumption for finding a suitable hyperelastic model is that torsion straining of the cylindrical surface is analogous to the straining in a simple shear. In search of deformation energy using hyperelastic models based on the first invariant (Yeoh, Fung and Yamashita-Kawabata) or on the main strains (Ogden, Nunes, Lopez-Pamies), which have been optimized for stress-strain curves for different temperatures and excitation frequencies. The polynomial Yeoh's 6-parameter model showed the best fit with the experiments. Furthermore, only important relationships for fitting these hyperelastic models are shown. It was considered to describe the deformation of Green's approach. Then constitutive relation for Cauchy stress is expressed in principal invariants as

$$\boldsymbol{\sigma} = -p\mathbf{I} + 2\frac{\partial W}{\partial I_1}\mathbf{B} - 2\frac{\partial W}{\partial I_2}\mathbf{B}^{-1}, \tag{1}$$

where p is the Lagrange multiplier also called hydrostatic pressure, which is associated with incompressibility, W is strain energy density function and $\mathbf{B}$ is left Cauchy Green strain. We assume herein a dependence of the strain energy density function W only on the first invariant I_1, i.e. $\mathrm{W} = \mathrm{W}(I_1)$. Then Cauchy stress tensor is expressed as

$$\boldsymbol{\sigma} = -p\mathbf{I} + 2\frac{\partial W}{\partial I_1}\mathbf{B}. \tag{2}$$

The Yeoh hyperelasticity model was chosen for a description of the density of deformation energy since Yeoh developed this model which depends on the first strain invariant only. For our case of a simple shear, his model with six parameters was the most suitable

$$W = \sum_{\mathrm{i}=1}^{6} C_{\mathrm{i}0}\left(I_1 - 3\right)^{\mathrm{i}}, \tag{3}$$

where W is a strain energy density, $C_{\mathrm{i}0}$ are hyperelastic constants and I_1 an invariant of the left Green strain tensor. After substitutions and derivations of the Cauchy shear stresses, equations 2 and 3 were arranged into the forms

$$\sigma_{12} = 2\gamma\left(C_{10} + 2C_{20}\left(I_1 - 3\right) + 3C_{30}\left(I_1 - 3\right)^2 + 4C_{40}\left(I_1 - 3\right)^3 + 5C_{50}\left(I_1 - 3\right)^4 + 6C_{60}\left(I_1 - 3\right)^5\right), \tag{4}$$

The constants of Yeoh model were evaluated from the experimental shear stress-strain curve using the least square method (LSM) in the form

$$\mathbf{c} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{t}, \tag{5}$$

where the vector $\mathbf{c} = \left[C_{10},\ldots,C_{60}\right]^{\mathrm{T}}$ represents the unknown constants of Yeoh model with six parameters, matrix A consists of coefficients of σ_{12} (4) for different values of measured γ and $\mathbf{t} = \left[\sigma_1,\ldots,\sigma_{\mathrm{n}}\right]^{\mathrm{T}}$ is the vector of experimental shear stresses. (Šulc et al., 2018) The calculated strain energy W obtained from (3) using the tuned constants $C_{\mathrm{i}0}$ for given three temperatures shows a weak quadratic dependence on strain is shown in Fig. 3 a). By increase of temperature the deformation energy grows faster in dependence on strains due to rubber material softening. Then the constant β of the HPD model is obtained from the proposed relationship between dissipated and deformation energy, which is expressed as follows

$$W_{\mathrm{Dis}} = \beta \cdot \omega \cdot W_{\mathrm{Def}}\ , \tag{6}$$

where β is the searched constant, ω is the excitation frequency, W_{Dis} the dissipated energy which is obtained from the hysteresis loop for one period of harmonic load and W_{Def} is the deformation energy determined from the shear stress analysis on the surface of the cylinder considering hyperelasticity. The dependence of this constant β on the strain for selected temperatures (20 ° C, 0 ° C, -20 ° C) is shown in Fig. 3 b). Here, the deformation region from 5% to 25% of the strain can be seen, where β is almost constant, which gives the applicability of the same HPD model within the whole range of mentioned strains, temperatures and the frequency 4 Hz.

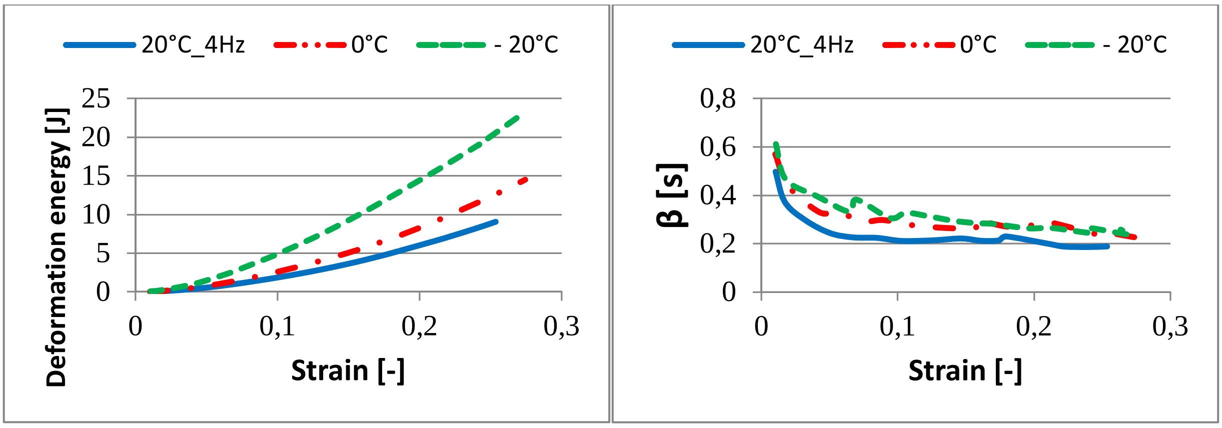

Fig. 3: a) Dependency of Dissipated energy vs. strain b) Dependency of the constant β vs. strain for temperature 20° C (solid line), 0° C (dot-and-dash line) and -20° C (dashed line) – type of lines is the same for a) and b).

4. Conclusions

In this paper, we dealt with temperature dependences of dissipated energy of hard rubbers at torsional finite deformations. The evaluated deformation and dissipation energies based on experiment were used for determination of constant β of HPD model. The value of constant β remains almost same in wide range of temperature and finite deformations. It shows that the HPD model with a given value of constant β is applicable for description of dissipation behavior in wide ranges of deformations and temperatures.

The proposed model can be used as the first approximation of the damping behavior of hard rubber elements for the dynamic loading.

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