STABILITY OF BEAMS UNDER UNIFORM LOADING \( q \) COMBINED WITH END MOMENTS \( M_a \) AND \( M_b \)

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Abstract: Calculation of the critical moments \( M_c \) of the beams under various combinations of the uniform loading \( q \) and the end moments. Solution of the differential equations is presented in the graphical form. Two diagrams enable to obtain the factors \( C_1 \) and \( C_2 \), which depend on the parameters \( \bar{M} \) and \( \psi \). Approximate formula for calculation of the critical moments depending on three factors \( C_1, C_2, C_3 \) and three dimensionless parameters \( z, g, \zeta \) was derived in (2000a). The values of the factors \( C_1, C_2, C_3 \) for basic loadings of the beams with the various combinations of the boundary conditions in vertical and horizontal bending and torsion for the monosymmetric I-sections were published in (2000b). They were later accepted for Eurocode (2007) and many national annexes. The presented results are valid for the beams simply supported with double symmetric I-sections. They may be used also for the continuous beams being slightly on the safe side. Numerical examples show that our results equal to the exact results of commercial computer programs. The procedure gives quickly practically exact results and therefore it is efficient tool for designers.

Keywords: Stability of beam in bending, Critical moment, Uniform loading combined with end moments, Double symmetric I-section.

1. Application of proposed procedure

Numerical example 1:

Input values: beam with span \( L = 6 \text{m} \) simply supported in vertical bending \( (k_z = 1) \), horizontal bending \( (k_t = 1) \) with rolled section IPE 200 is loaded by uniform loading \( q = 5 \text{kN/m} \) acting on the surface of the upper flange \( (z_g = +100 \text{mm}) \). The values of the end moments are \( M_a = -9,6 \text{kNm} \) and \( M_b = 0 \text{kNm} \) (Fig.1). \( E = 210 \text{GPa}, G = 81 \text{GPa}, S = 235, f_y = 235 \text{MPa}, \gamma_{M0} = 1,0, \gamma_{M1} = 1,0 \). Cross-sectional properties of IPE 200 – DIN 1025, Part 5 (03/1994): \( h = 200 \text{mm}, b = 100 \text{mm}, t_t = 8,5 \text{mm}, t_w = 5,6 \text{mm}, \) buckling curve “a”, cross-section Class 1, \( I_z = 1,943 \text{cm}^4, W_{pl,y} = 220,64 \text{cm}^3, M_{pl,y,Rk} = 51,85 \text{kNm}, L_z = 142,4 \text{cm}^4, I_w = 12,746 \text{cm}^6, I_t = 6,846 \text{cm}^4 \). For the double symmetric section, the parameter of section symmetry \( z_j = 0 \text{mm} \), the factor \( C_3 \) is not necessary. \( M_{fd,max} = 17,955 \text{kNm} \) is in the section \( x = 3,3 \text{m} \).

Numerical example 2:

Input values: the same as in the example 1, only difference is that point load of application of the uniform loading \( q \) is on the surface of the bottom flange of IPE 200 section \( (z_g = -100 \text{mm}) \). The results of the example 2 are given below in the brackets. Dimensionless parameters of the beam:

\[
\zeta = \frac{\pi a}{k_z L}, \quad \gamma = \frac{\pi a}{k_t L}, \quad \zeta_j = \frac{\pi j}{k_z L}, \quad \gamma_j = \frac{\pi j}{k_t L}
\]

\[
\kappa_w = \frac{\pi}{k_z L} \sqrt{\frac{E I_z}{G I_t}} = 0.364, \quad \zeta_g = \frac{\pi a}{k_z L} \sqrt{\frac{E I_z}{G I_t}} = 0.385 (-0.385), \quad \zeta_j = \frac{\pi j}{k_z L} \sqrt{\frac{E I_z}{G I_t}} = 0
\]

(1)

a) Calculation of the critical moment \( M_c \) of the ideal beam:

Diagram parameters \( \bar{M} \) and \( \psi \): \( |M_a| = -9,6 \text{kNm} | > |M_b| = 0 \text{kNm} \) \( \rightarrow M = M_a = -9,6 \text{kNm} \), \( M_q = qL^2/8 = 5 \text{kN/m} (6 \text{m})^2/8 = 22,5 \text{kNm} \). For parameters \( \psi = M_b / M_a = 0 \), \( \bar{M} = M / (|M| + M_q) = \)

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we obtain the factors $C_1 = 1.18$, $C_2 = 0.59$ from Figs. 1, 2.

$$\mu_{cr} = \frac{C_1}{k_2} \left[ \sqrt{1 + \kappa^2_{cr} + \left(C_2 \varepsilon_g + C_3 \varepsilon_f \right)^2} - \left(C_2 \varepsilon_g + C_3 \varepsilon_f \right) \right] = 1.016, \ (1,552)$$

(2)

Critical moment: 

$$M_{cr} = \mu_{cr} \frac{\pi \sqrt{EI_z GI}}{L} = 21,667 \text{kNm}, \ (33,082 \text{kNm})$$

(3)

Critical moment calculated by LTBeam: 

$$M_{cr, \text{LTBeam}} = 21,568 \text{kNm}, \ (32,799 \text{kNm})$$

(4)

b) Verification of the real beam:

relative slenderness 

$$\bar{\lambda}_{LT} = \frac{W_{pl,y} f_y}{M_{cr}} = 1.547, \ (1,252), \ \text{imperfection factor} \ \alpha_{LT} = 0.21,$$

(5)

$$\Phi_{LT} = 0.5 \left[ 1 + \alpha_{LT} \left( \bar{\lambda}_{LT} - 0.2 \right) + \frac{\bar{\lambda}^2_{LT}}{\bar{\lambda}^2} \right] = 1.838, \ (1,394), \ \chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi^2_{LT} - \bar{\lambda}^2_{LT}}} = 0.353, \ (0,498).$$

(6)

Design resistance of the beam: 

$$M_{b, Rd} = \chi_{LT} W_{pl,y} \frac{f_y}{\gamma_M} = 18,319 \text{kNm}, \ (25,828 \text{kNm}).$$

(7)

Utilization factor:

$$U = \frac{M_{Ed}}{M_{b, Rd}} = \frac{17,955}{18,319} = 0.98 < 1.0, \ (0.695 < 1.0).$$

(8)

### Tab. 1: Bending moment distributions as function of the parameters $\bar{M}$ and $\psi$

<table>
<thead>
<tr>
<th>$\psi - \frac{M_a}{M_b}$</th>
<th>$\bar{M} = \frac{\mu_{cr} \sqrt{EI_z GI}}{L}$</th>
<th>$\bar{M}-1$</th>
<th>$-1 &lt; \bar{M} &lt; 0$</th>
<th>$\bar{M} = 0$</th>
<th>$0 &lt; \bar{M} &lt; 1$</th>
<th>$\bar{M}=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi = 1$</td>
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References


\[ M = \frac{M}{|M| + \frac{1}{8} qL^2}, \quad M = M_a \text{ if } |M_a| \geq |M_b| \text{ otherwise } M = M_b, \quad -1 \leq \psi \leq 1, \quad \frac{h}{2} \leq z_c \leq \frac{h}{2}. \]
Fig. 2: Factor $C_2$ for calculation of critical moment $M_{cr}$ of simply supported beam with double symmetric I-section as function of parameters $\overline{M}$ and $\psi$.