APPLICATION OF WAVES SPECTRAL ANALYSIS FOR DETERMINATION OF GROUND DYNAMIC PARAMETERS

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Abstract: Assessment of soil dynamic parameters procedure utilizing results of elastic or viscous–elastic wave analysis is shortly described. For the ISM experimental study of ground stress–wave propagation permits the ground to be modelled as a damped, viscoelastic half space. The viscoelastic model of soil simulation using the complex modulus conception offers a very good approach to the actual soil behavior. The paper also presents the experimental ISM tests procedure in-situ and the results obtained by correlation and spectral analysis application. The proposed approach also is applicable to analyze of the shear and surface waves due to another sources of vibration on the ground half-space (e.g., microtremor waves).

Keywords: Soil body wave propagation process, Application experimental impulse–seismic methods ISM, In-situ tests, Correlation and spectral analysis, FEM modeling.

1. Introduction

The constitutive response of soil under dynamic loading is complex. Laboratory tests show that the soil behavior is anisotropic and non-linear. For less cohesive dry soils, the non-linear soil behavior can be neglected when the shear strain \( \gamma \) is smaller than \( 10^{-5} \). This is the case for the vibrations in the free field induced by human activities (microtremor), e.g., railway and road traffic, construction works, etc. The basic theoretical equations used to describe the viscoelastic half-space analysis of the Raleigh’s and shear waves propagation through the ground with a modulus in a complex form is described, e.g., by Biot (1956). The experimental tests for the purpose of the evaluation of elastic and attenuation soil parameters are performed at the tests site, e.g., in (Schevenels, 2004). The wave velocities determination based on the spectral analysis theories can be employed to determine body wave velocities in the in-situ ISM test. In microtremor processes analyses the test and the theory data combination also enables to calculate the prediction level of ground vibration, see, e.g., (Kolsky, 1960; Benčat, 2009).

2. Methods

The Impulse seismic method (ISM) aims to determine the dynamic shear modulus and the damping parameters of the half-space shallow soil depths where microtremor vibrations are dominant. The shear wave velocity and material damping parameter are important for calculation of vibrations in the free field or in buildings due to road or rail traffic, industrial machinery, construction activities, etc. The ISM is based on the in-situ experiments where the Rayleigh waves are generated by means of a falling weight or impact hammer. The resulting wave field is recorded by a number of sensors on the soil’s surface and used to determine the shear or Rayleigh wave velocity and soil attenuation parameters (material and geometrical damping ratios) of the soil.

2.1 The impulse test description

The common practice of the ISM uses a linear source-receiver array with two or more receivers located at distance \( l \) (m) from the source. The propagation of body waves generated by the source is monitored with the receivers at the same depth as the source, (e.g., A1, A2 and A3, see Fig. 1). The traditional approach

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used in the in-situ tests to determine shear wave velocity ($v_s$) is based on identifying the time interval of the wave travelling between, e.g., the A1 and A3 receivers. Once these times are determined, velocities are calculated via dividing distance ($l$) by appropriate times. The wave velocities determined by source to receiver measurements are termed direct velocities method. Other techniques based on correlation and spectral analysis theories (as well as other seismic tests like cross-hole and down-hole test, etc.) can be employed to determine the body and surface wave velocities as in the ISM test. This approach offers benefits in two areas. First, interval velocities have fewer potential errors than direct velocities. Second, the techniques can be fully automated (Bendat, 1993). Furthermore, the retrieval of additional information such as the strain rate effects and the material damping is possible with the spectral analysis as well. The first of these time–determination techniques is based on the cross-correlation function of waves travelling by two receivers. The cross-correlation $R_{xy}(\tau)$ of two functions $w_1(t)$ and $w_2(t)$ is given by the integral:

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} w_1(t)w_2(t + \tau)dt,$$

where $w_1(t)$ and $w_2(t)$ are time histories of the motion of the wave passing by the first and the second receivers (waveforms); $\tau$ is the time delay and $t$ is the variable of integration. The two waveforms to be correlated are the same but one lags the other a time $t^\ast$. If these two shifted but otherwise identical waveforms represent the waves recorded at two receivers, the time $t^\ast$ will correspond to the time at which the peak of the cross-correlation function occurs and will represent the travel time of the wave between the receivers. If the distance between receivers is divided by time, the $v_s$ velocity will be obtained. Another way to calculate wave velocities is based on the cross-energy spectral density (CESD) functions of the waveforms obtained at two different receivers (A1, A3). The CESD $G_{xy}(f)$ of two functions, $w_1(t)$ and $w_2(t)$, is:

$$G_{xy}(f) = \overline{G(f)}H(f),$$

where $G(f)$ and $H(f)$ are the Fourier transforms of $w_1(t)$ and $w_2(t)$ respectively, $f$ is frequency in Hz and \(\overline{\cdot}\) indicates the complex conjugate. Assume that the functions $w_1(t)$ and $w_2(t)$ are time records at the two receivers for each frequency, the phase of the cross-energy spectrum of $w_1(t)$ and $w_2(t)$ will give the phase difference of the corresponding harmonic. Since the time period of that harmonic is known ($T = 1/f$), the travel time between receivers can be obtained for each frequency by:

$$t(f) = \frac{\Theta_{xy}(f)}{2\pi f},$$

where $\Theta_{xy}(f)$ is the phase of the cross-energy spectrum (in radians). The distance between receivers ($l$) is known parameter. Therefore, the apparent velocity is:

$$v_s(t) = \frac{l}{t(f)}.$$

### 2.2 In situ experimental tests description

The experimental ISM tests in situ are performed by the impact device (Light Falling Weight Device - LFWD) depicted in Fig. 1. The LFWD consists of the circular rigid plate (1) with the contact area $A = 1000.0 \text{ cm}^2$, dropping weight (2) with mass $Q = 12.5 \text{ kg}$, indentation for setting the height of the weight (3), springs (4), plunger (5) guide rod (6) casing (7) and safety pin (8).

In each performed dynamic test, there are usually carried out 6 impulses in the measured surface area (spot) caused by dropping weight from constant height $h$. The height $h$ was set experimentally to achieve the constant area impact stress $p = 0.22 \text{ MPa}$. This stress value is usually requested for in situ soils dynamic and static loading tests, (DLT and SLT) in highway or railway construction engineering (subbase, roadbed, ground, etc.). In the ISM common practice, the dynamic impulse is carried out by LFWD into the half-space and then arose impact stress field is propagated throughout the half-space mainly as a surface wave and is monitored via receivers ($A_i, A_{i+1}$) at the same depth as the impulse source.

### 2.3 Case study

To calculate the prediction vibration level and the dynamic response for buildings in Bratislava urban areas near the tram track after its general reconstruction, it was necessary to know the soils dynamic parameters
and the transfer function (TF) or the frequency response function (FRF) in these sites. Therefore, the *in-situ impulse seismic method* (ISM) tests in the relevant areas were performed by Benčat (1992). The relevant quantities were obtained using the procedure described in Section 2.1, according to (Benčat, 1993; Benčat, 2014).

The example of ISM application is described by test process and its results at residence building (RB) No.B8 of the building site situated nearby the tram line *Vajnory Radial Line in Bratislava* (sandy loam - 3.5 m and gravel sand - 12.0 m). This local geology situation permits the ground to be modelled as a damped, viscoelastic half-space. The viscoelastic model of the soil simulation using the complex modulus conception $E^* = E(1 + \delta\epsilon)$ and $G^* = G(1 + \delta\epsilon)$, respectively, offers a very good approach to the actual soil behavior ($E$, $G$ and $\delta\epsilon \approx \delta\sigma$ are the real and imaginary components of complex modulus). The Raleigh’s and shear waves propagation $v_R$ and $v_S$ in the half-space with this form are analyzed, e.g., by Martincek (1994). The experimental tests for the purpose of the elastic and attenuation soil parameters evaluation are described in, e.g. (Benčat, 2008, 2014; SNA to Eurocode 8, 2010).

The *coefficient of attenuation* was historically derived from the Golitsin’s formula, (Golitsin, 1912), in the form:

$$\alpha = \left( l_y - l_0 \right)^{-1} \ln \left( k \sigma(0)/\sigma(y) \right), \quad k = \left( l_0/l_y \right)^{1/2}$$  \hspace{1cm} (5)

Modified formula (5) is defined also in STN EN 1998 - 1/NA/Z1, (SNA to Eurocode 8, 2010). In technical report (Benčat et al., 2014) is described the RB No.B8 building site in Bratislava with layout of accelerometers $A_c$ positions and impact loading ($I_L$) position during the tests series.

The impulse test results: $v_R = 145.10 \text{ ms}^{-1}$; $\delta\epsilon = 0.117$; $E_0 = 109.20 \text{ MPa}$; $G_0 = 41.10 \text{ MPa}$. As an example of the ISM application, the time history $v(t)$ and the cross-correlation functions $R_{xy}(t)$ from No.B8.6 test are plotted in Fig. 2.

The calculation includes the following data: $\lambda_s = 9.2 \text{ m}$, (Raleigh’s wave length); $p = 1950 \text{ kgm}^{-3}$, (soil mass density); $\alpha = 0.0398 \text{ m}^{-1}$. The attenuation coefficient $\alpha$ was obtained by application of standard deviations $\sigma(0)$, $\sigma(y)$ of the displacement amplitude vibration at the distance $l_0$, $l_y$ from the source of excitation using the displacement energy spectral densities functions $G_{ii}^{00}$ and $G_{kk}^{00}$. The used procedure represents the energy approach for determining of the dynamic shear modulus and the damping parameters in-situ via application of cross-energy spectral density functions and cross-correlation functions for the given frequencies range.

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Fig. 1: Light Falling Weight Device: (a) scheme, (b) view in situ.

Fig. 2: The ISM test RB - 8.6 correlation analysis results at points A1, A2 and A4 example.
3. Conclusions

The impulse seismic method aims to determine the dynamic shear modulus and the damping parameters of the shallow soil half-space depths where microtremor vibrations are dominant. The shear wave velocity and material damping are important for calculation of the free field or buildings vibrations due road or rail traffic, industrial machinery, construction activities, etc.

In situ tests preserve the natural status of the soil and avoid the sample disturbance which can arise in laboratory tests. Moreover, a larger volume of the soil is examined when performing in situ tests, avoiding a bias in the results due to local variations of the soil properties.

The ISM is based on the hypothesis that the response of the soil in the ISM test is due to a single surface wave with dominant energy and frequency. The occurrence of multiple Rayleigh modes does not affect the attenuation parameters of either the fundamental or the dominant Rayleigh wave, as all modes appear as separate, non-interfering.

The soil attenuation parameters were determined by standard deviations $\sigma(0)$, $\sigma(y)$ of velocities amplitude vibration at points in the distances $l_0$, $l_1$ from the source of excitation using the velocity ESD $G_0(0)$ and $G_{ik}(y)$, which represent the surface waves energy at these points. The waves travel time determined by the correlation analysis required to calculate shear wave velocities from which initial tangent moduli $(G_0, E_0)$ were determined. Soil attenuation parameters ($\alpha$) were determined by standard deviations calculated from energy spectral densities of the traveled waves at measured surface points.

This paper also presents some results of the in-situ experimental ISM tests procedure and results obtained by correlation and spectral analysis.

The used procedure represents the energy approach for determining the dynamic shear modulus and the damping parameters in-situ via application of the cross-energy spectral density functions and cross-correlation functions for a given frequencies range of the vibration processes.

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