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ANALYTICAL AND EXPERIMENTAL ANALYSIS OF STRESS UNIFORMITY IN SPECIMEN DURING DIRECT IMPACT HOPKINSON BAR TEST

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Abstract: *Stress uniformity in a specimen represents a crucial parameter for validity of impact experiment conducted in a split Hopkinson bar. Principle of the experimental method is to measure elastic stress waves that propagate in the bars of an experimental setup and to calculate stress to strain relations at the interfaces with the specimen. However, standard procedure to evaluate the experiment can only be valid when stress is distributed approximately uniformly along the specimen. As the stress wave has to propagate through the specimen during the experiment, a time period of significantly non-uniform stress distribution is present and a certain number of wave transits is required to achieve approximately uniform stress distribution. In this paper, a simple analytical model assuming one-dimensional wave propagation theory, non-equal mechanical impedance of the bars and linear elastic material model of the specimen is introduced to determine a number of transits required to achieve uniform stress distribution in the specimen. Potential of the calculated results is discussed in an experiment using a direct impact Hopkinson bar and a material with significant plateau in its stress-strain response.*

Keywords: Split Hopkinson bar, Direct impact testing, Specimen stress uniformity, Wave propagation.

1. Introduction

A split Hopkinson bar is an experimental technique for impact testing of materials. In this technique, a specimen is placed between two bars and subjected to either high strain rate compression or tension by direct or indirect impact. Principle of the experimental method is based on propagation of elastic stress waves in the bars of the experimental setup. The elastic stress waves are measured and the signals are used for evaluation of force and displacement relations at the interfaces with the specimen. As the stress wave has to propagate through the specimen during the impact, the stress distribution along the specimen is non-uniform. This effect is most significant at the beginning of the impact when the stress wave has to propagate through the specimen for the first time. Then, the stress distribution converges to an approximately uniform state with the increasing number of wave transits. The rate of the convergence depends on mechanical impedance of either the bars and the specimen.

The stress uniformity and convergence of dynamic forces represent a topic that is highly investigated as it affects validity of the experiments conducted using a split Hopkinson bar method (Jakkula et al., 2022). Analytical models have been developed to analyze the stress uniformity in the specimen and the rate of dynamic forces convergence with one of the most important model published by Yang et Shim (2005). In this paper, an analytical model assuming the one-dimensional wave propagation theory and a conventional split Hopkinson bar setup with the identical bars is introduced. Using the model, a required number of wave transits to reach stress difference lower than $5\,\%$ between the input and output interface of the specimen can be calculated. The paper also investigates an influence of the wave shape of the initial wave propagating through the specimen, concluding that stress uniformity occurs much sooner in the case of a wave with linear ramp (Yang et Shim, 2005). The aforementioned paper represents an important overview of the stress increasing mechanism in the specimen during the impact and discusses validity of split Hopkinson bar experiments.

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Nowadays, so-called direct impact Hopkinson bar methods have been increasingly used to investigate the dynamic behavior of low impedance cellular materials and metamaterials during dynamic impact (Jakkula et al., 2022). For this type of materials, the convergence of dynamic forces is a crucial factor. To achieve better quality of the strain signals in the case of the direct impact Hopkinson bar, different mechanical impedance of the input and the output bars is often used. Therefore, the analytical model published by Yang et Shim (2005) was in this paper modified to account for direct impact scenario with different mechanical impedance of the bars. The model cannot account for complex wave propagation effects in metamaterials, however, it represents an useful tool for estimation of force convergence period and for analysis of the experiment validity. In the paper, the results of the model are compared with the experimental results of wave propagation in a real specimen with a significant plateau in its stress-strain curve.

2. One-dimensional wave propagation and stress uniformity model

The model is assuming the infinite linear elastic bars while the specimen is represented by a linear elastic material model and placed between the bars. Mechanical impedance of the specimen is lower than that of either the bar. The impact is induced by a collision of the input bar with the specimen at the initial impact velocity. In the following list, main parameters of the model are summarized.

Nomenclature

Δt	time required for the stress wave to travel from one side of the specimen to another
ρ_1	density of the output bar
ρ_2	density of the input bar
ρ_s	density of the specimen
A_1	area of the output bar
A_2	area of the input bar
A_s	area of the specimen
c_1	wave propagation velocity in the output bar ($\sqrt{E_1/\rho_1}$ for ideal linear elastic material)
c_2	wave propagation velocity in the input bar ($\sqrt{E_2/\rho_2}$ for ideal linear elastic material)
c_s	wave propagation velocity in the output bar ($\sqrt{E_\mathrm{s}/\rho_\mathrm{s}}$ for ideal linear elastic material)
E_1	Young's modulus of the output bar
E_2	Young's modulus of the input bar
E_s	Young's modulus of the specimen
k	number of wave transits through the specimen-to-bar interface
v_0	initial impact velocity of the input bar

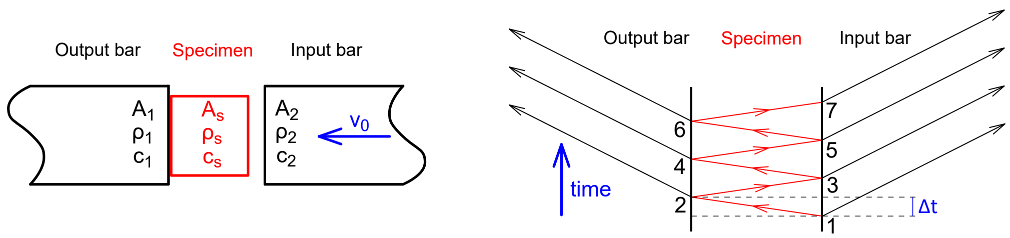

Fig. 1: Definition of the analytical model (left), Langrange diagram of wave propagation (right).

The definition and main parameters of the analytical model together with Lagrange diagram of wave propagation during the impact are shown in Fig. 1. The model is based on the one-dimensional wave propagation theory without wave dispersion effects. During the contact of the input bar with the specimen, a stress wave is developed right at the interface between the specimen and the input bar. The stress wave propagates through the specimen and is partially reflected at the interface with the output bar. The reflections of the stress wave at the interfaces between the bars and the specimen cause an increase in stress in the specimen. The stress distribution starts to be more uniform with the increasing number of wave transits through the specimen. The model works with different mechanical impedance of either the bars and the specimen. Two scenarios are taken into account: i) ideal elastic collision creating a perfect rectangular stress pulse with unlimited duration, and ii) elastic collision creating a stress wave with linear ramp followed by a rectangular stress pulse of unlimited duration. Lagrange diagrams showing the stress to velocity dependence during the impact for both types of the stress wave are shown in Fig. 2.

Based on the schemes shown in Fig. 2, stress increments $\Delta\sigma_\mathrm{k}$ corresponding to a k^th transition of the stress wave through the interface can be calculated. In the case of a rectangular pulse, the stress increments for the first transition can be calculated using the velocity increments at the input interface d_ik and the output interface d_ok. The velocity and stress increment for $k = 1$ are expressed by

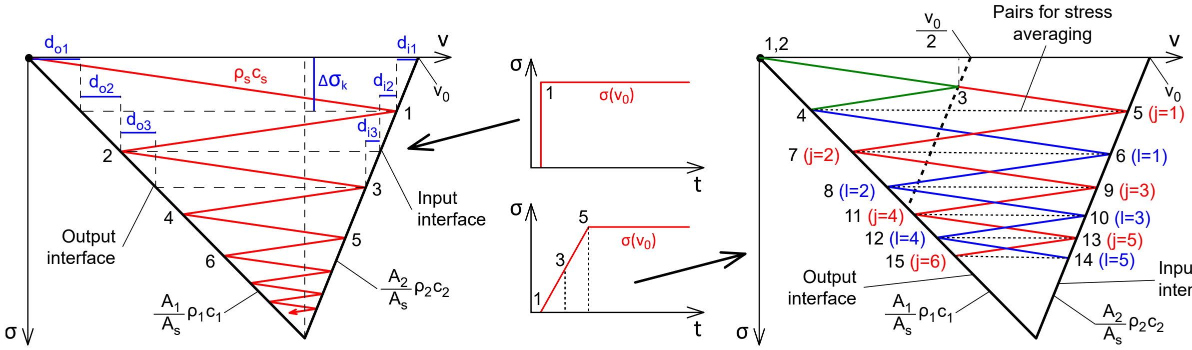

Fig. 2: Lagrange diagram showing the stress to velocity dependency during the impact for a perfectly rectangular pulse (left) and a pulse with linear ramp period with duration of $2\Delta t$ *(right).*

$$d_{\mathrm{i}1} = \frac{A_\mathrm{s}\rho_\mathrm{s}c_\mathrm{s}v_0}{(A_2c_2\rho_2 + A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s})},\quad d_{\mathrm{o}1} = \frac{A_\mathrm{s}\rho_\mathrm{s}c_\mathrm{s}\left(v_0 - \dfrac{A_\mathrm{s}\rho_\mathrm{s}c_\mathrm{s}v_0}{A_2c_2\rho_2 + A_\mathrm{s}\rho_\mathrm{s}c_\mathrm{s}}\right)}{A_1\rho_1c_1},\quad \Delta\sigma_1 = \frac{A_2c_2\rho_2}{A_\mathrm{s}}\cdot d_{\mathrm{i}1}. \tag{1}$$

Using the analogical approach, the stress increment for arbitrary transition can be calculated. The increments have to be evaluated separately for the input (odd k) and the output (even k) interface using relations

$$\begin{aligned}\Delta\sigma_{\mathrm{k(in)}} &= |A_2c_2\rho_2\rho_\mathrm{s}c_\mathrm{s}v_0\left(A_2c_2\rho_2 - A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{k-1}{2}}\left(A_1c_1\rho_1 - A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{k-1}{2}}\cdots\\ &\cdots\left(A_2c_2\rho_2 + A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{-k-1}{2}}\left(A_1c_1\rho_1 + A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{-k+1}{2}}|; k\in(1,3,5,\cdots), \qquad (2)\\ \Delta\sigma_{\mathrm{k(out)}} &= |A_2c_2\rho_2\rho_\mathrm{s}c_\mathrm{s}v_0\left(-A_2c_2\rho_2 + A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{k-2}{2}}\left(A_1c_1\rho_1 - A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{k}{2}}\cdots\\ &\cdots\left(A_2c_2\rho_2 + A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{-k}{2}}\left(A_1c_1\rho_1 + A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{-k}{2}}|; k\in(2,4,6,\cdots). \qquad (3)\end{aligned}$$

For the rectangular stress pulse, the actual stress in the specimen can be calculated by summing up the individual increments while the difference between the input and the output interface is equal to the actual stress increment. Relative stress uniformity U can be thus investigated through relations

$$\sigma = \sum_{\mathrm{j}=1}^{\mathrm{k}}\Delta\sigma_\mathrm{j},\quad U = \frac{\Delta\sigma_\mathrm{k}}{\sigma}. \tag{4}$$

In case of the stress pulse with the linear ramp, the problem can be divided into three sections: i) initial linear ramp-in phase $k = 1, 2, 3, 4$ (the green curve in the right diagram in Fig. 2), ii) stress wave propagation of the initial part of the pulse ($k > 4$ for the blue curve in the right diagram in Fig. 2), iii) propagation of the rectangular stress wave ($k \geqq 5$ for the red curve in the right diagram in Fig. 2). For phase iii), Eq. 2 and Eq. 3 can be used when variable j is substituted for k and corresponds to $j = 1$ when $k = 5$ (see red numbers for j in the right diagram in Fig. 2). For phase i), the following relations for stresses can be used

$$\sigma_3 = \frac{A_2c_2\rho_2\rho_\mathrm{s}c_\mathrm{s}v_0}{2A_2c_2\rho_2 + 2A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}},\quad \sigma_4 = \frac{A_2c_2\rho_2\rho_\mathrm{s}c_\mathrm{s}v_0A_1c_1\rho_1}{\left(A_2c_2\rho_2 + A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)\left(A_1c_1\rho_1 + A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right).} \tag{5}$$

For phase ii), the stress increments $\Delta\sigma_\mathrm{l}$ can be calculated using relations (for l see blue numbers in the right diagram in Fig. 2)

$$\begin{aligned}\Delta\sigma_{\mathrm{l(in)}} &= |A_2c_2\rho_2\rho_\mathrm{s}c_\mathrm{s}v_0\left(A_1A_2c_1c_2\rho_1\rho_2 + A_\mathrm{s}^2c_\mathrm{s}^2\rho_\mathrm{s}^2\right)\left(A_2c_2\rho_2 + A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{-l-3}{2}}\cdots\\ &\cdots\left(A_1c_1\rho_1 + A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{-l-1}{2}}\left(A_2c_2\rho_2 - A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{l-1}{2}}\left(A_1c_1\rho_1 - A_\mathrm{s}c_\mathrm{s}\rho_\mathrm{s}\right)^{\frac{l-1}{2}}|; l\in(1,3,5,\cdots), \qquad (6)\end{aligned}$$

$$\Delta\sigma_{\mathrm{l(out)}} = |\rho_{\mathrm{s}} c_{\mathrm{s}} v_0 \left(A_1 A_2 c_1 c_2 \rho_1 \rho_2 + A_{\mathrm{s}}^2 c_{\mathrm{s}}^2 \rho_{\mathrm{s}}^2\right) \left(A_2 c_2 \rho_2 + A_{\mathrm{s}} c_{\mathrm{s}} \rho_{\mathrm{s}}\right)^{\frac{-l-2}{2}} \cdots$$
$$\cdots \left(A_1 c_1 \rho_1 + A_{\mathrm{s}} c_{\mathrm{s}} \rho_{\mathrm{s}}\right)^{\frac{-l-2}{2}} \left(-A_2 c_2 \rho_2 + A_{\mathrm{s}} c_{\mathrm{s}} \rho_{\mathrm{s}}\right)^{\frac{l-2}{2}} \left(A_1 c_1 \rho_1 - A_{\mathrm{s}} c_{\mathrm{s}} \rho_{\mathrm{s}}\right)^{\frac{l}{2}} |; l \in (2, 4, 6, \cdots). \quad (7)$$

Then, two series with the stress increments exist, $\Delta\sigma_{\mathrm{C}} = (0_{\mathrm{k=1}}, \sigma_{3,\mathrm{k=3}}, \Delta\sigma_{\mathrm{j=1,k=5}}, \Delta\sigma_{\mathrm{j=2,k=7}}, \cdots)$ for part with the constant pulse (odd k) and $\Delta\sigma_{\mathrm{L}} = (0_{\mathrm{k=2}}, \sigma_{4,\mathrm{k=4}}, \Delta\sigma_{\mathrm{l=1,k=6}}, \Delta\sigma_{\mathrm{l=2,k=8}}, \cdots)$ for the part covering the pulse with the linear ramp (even k). Stress increments can be transformed to absolute stress values by summing up the increments in the stress increment series (see Eq. 4). The stress uniformity U can be then analyzed through average stress σ_{A} through relations

$$\sigma_A = \frac{1}{2}\left(\sigma_{\mathrm{k+1}} + \sigma_{\mathrm{k}}\right), \quad U = \frac{\sigma_{\mathrm{k+1}} - \sigma_{\mathrm{k}}}{\sigma_A}. \quad (8)$$

3. Results

The model was used to estimate the number of wave transits through a specimen in case of direct impact Hopkinson bar with the input bar made of aluminum alloy and the output bar made of polymethylmethacrylate (PMMA). The specimen was a cube of expanded polypropylene (EPP). Relative stress uniformity for case of the ideally rectangular pulse and the pulse with a linear ramp is shown in Fig. 3 on the left. Note that stress uniformity was, according to the model, reached much faster in the case of the pulse with a linear ramp (similarly to the results in (Yang et Shim, 2005)) when only 4 interface transits (approx. $40\,\mu$s) were required. The measured stress time curves are shown in Fig. 3 on the right. Note that because the material exhibited a profound plateau region, which can be, with approximation, interpreted as a change from linear elastic to perfect plastic behavior (not increasing stress) in the model, the stress equilibrium was reached very quickly, almost instantly after the first transition of the stress wave.

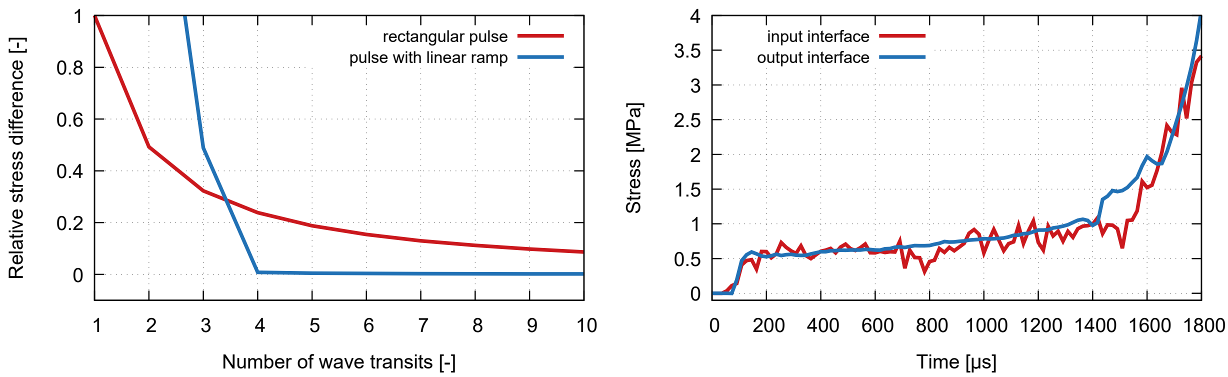

Fig. 3: Relative stress uniformity predicted by the model (left). Stress to time experimental data (right).

4. Conclusions

The existing analytical model describing the development of stress uniformity in a specimen during the split Hopkinson bar experiment was modified to account for a scenario with the bars with different mechanical impedance, an arrangement often used in direct impact Hopkinson bar setups. Similarly to the already published results, the model predicted a quick convergence of the input and output forces in a case of an impact producing a stress wave with a linear ramp. The output of the model was compared with the experiment showing that for materials with significant plateau region, the stress convergence develops very quickly with a time period corresponding to a period in a pulse variant with a linear ramp.

Acknowledgments

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References

Jakkula, P. et al. (2022) The Symmpact: A Direct-Impact Hopkinson Bar Setup Suitable for Investigating Dynamic Equilibrium in Low-Impedance Materials. *Experimental Mechanics*, Vol. 62, pp. 213–222.

Yang, L.M., Shim, V.P.W. (2005) An analysis of stress uniformity in split Hopkinson bar test specimens. *International Journal of Impact Engineering*, Vol. 31, No. 2, pp. 129-150