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STOCHASTIC MODELLING OF OVERFLOW FROM A SPINNING CONTAINER

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Abstract: *Abstract: This paper introduces a stochastic approach to the solution of fluid mechanics. A cylindrical container filled with a liquid rotates about its vertical axis and the problem of the probability of overflow of the container and the calculation of the hydrostatic pressure are solved. The calculations were performed in the Anthill program using the Monte Carlo method. For the stochastic calculations, the input parameters of the mathematical model were assigned deviations using probability distribution histograms reflecting the real design and operating conditions. The obtained results can be used to evaluate the reliability of a technological process where rotating fluids are present.*

Keywords: Probability, Simulation-Based Reliability Assessment (SBRA) method, Probabilistic reliability assessment, Spinning container overflow, Hydrodynamics.

1. Introduction

The issue of overflowing of a rotating vessel is generally an undesirable phenomenon in technical operation. This paper focuses on the mechanics of fluid in a rotating vessel. New stochastic approaches (Monte Carlo method, Simulation-Based Reliability Assessment - SBRA method) are used to determine the probability of vessel overflow and to calculate the hydrostatic pressure. The obtained knowledge can be used in the design, innovation of rotating vessel or reliability assessment of technological process using rotating fluids. Probabilistic approaches or probabilistic reliability assessments are nowadays innovatively applied in many fields of science and technology, see e.g. (Frydrýšek et al. 2012; Frydrýšek et al. 2013; Grepl et al. 2014; Frydrýšek et al. 2018) and (Bajtek et al. 2020)

2. Theory

A cylindrical container of inner radius R and height H is filled with liquid to a height H_liq from the bottom of the container, see Fig. 1(a). When set in motion, the vessel rotates about its axis with speed n and the liquid level changes, see Fig. 1(b).

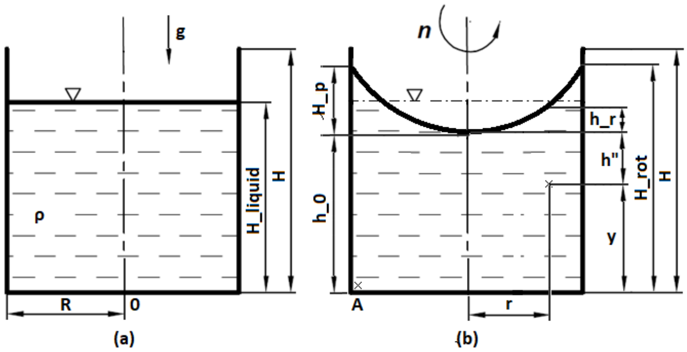

Fig. 1: Liquid container (a) at rest, (b) in rotation.

The basic knowledge of fluid mechanics about rotating vessels was used to program the calculation (Drábková, 2007). During rotation, a centrifugal force, represented by the centrifugal acceleration $r\omega^2$, acts on the fluid in the horizontal direction and a gravitational force, represented by the gravitational acceleration $-g$, in the vertical direction. The equation of the surface area under rotation, see Fig. 1(b), has the form

$$\frac{r^2\omega^2}{2} - g(y - h_0) = 0, \tag{1}$$

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which is the equation of the parabola. The height of the paraboloid H_p measured on the shell of the cylindrical container at radius r = R is determined as

$$H\_p = \frac{r^2\omega^2}{2g} \,. \tag{2}$$

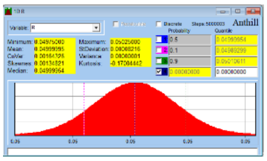
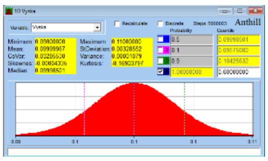
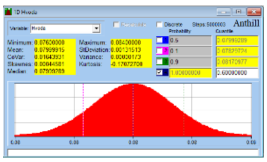
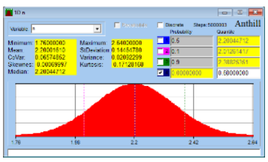
The maximum hydrostatic pressure is at location A, see Fig. 1(b), and is given by

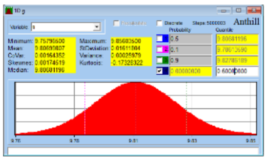
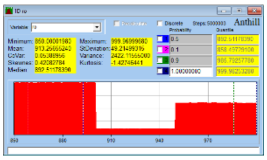
$$p = \rho g(h_r + h") = \rho g h \,. \tag{3}$$

3. Probabilistic Inputs

An innovative stochastic approach was used to solve whether the container overflows due to rotation. By respecting the real variability of the input variables, the probability with which overflow will occur was obtained. In order to perform the calculation, probabilistic inputs were defined using bounded histograms. The Anthill software (i.e. the SBRA Method) was used for the actual calculation, see Marek et al. 2003 and Grepl et al. 2014. Six mutually independent variables of the probabilistic inputs were selected. These random, or pseudo-random, variables represent real variables, and their variability/volatility represents production tolerances, different altitudes, or liquid with other substance admixture. In practice, the vessel contains various oils, water, or emulsions. Definitions of the histograms are given in Table 1 and Fig. 1, including basic statistics.

Tab. 1: Stochastic inputs and their basic characteristics in Anthill software

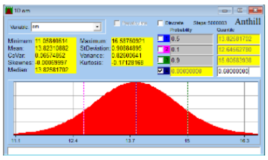
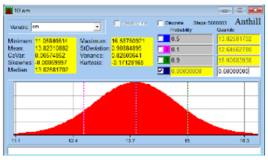
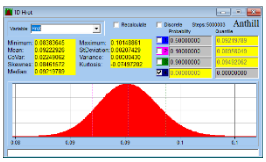
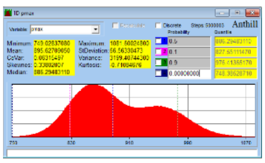
Random inputs	Description	Histogram applied in Anthill software	Min.	Max	Median	Mean	St. deviation
R /m/	Container radius	Modified (truncated) normal distribution 0.05*"n1-005.dis" 	0.04975	0.05025	0.4999954	0.04999995	0.00008216
H /m/	Container wall height	Modified (truncated) normal distribution 0.1*"n1-10.dis" 	0.09	0.11	0.0999850	0.09999967	0.00328552
H_liq /m/	Water level at rest	Modified (truncated) normal distribution 0.08*"n1-05.dis" 	0.076	0.084	0.0799928	0.07999915	0.00131513
n /min^{-1}/	Revolutions	Modified (truncated) normal distribution 2.2*"n1-20.dis" 	1.76	2.64	2.2004471	2.20001610	0.14464780

g /ms^{-2}/	Gravitational acceleration	Modified (truncated) normal distribution 9.807*"n1-005.dis" 	9.75796	9.85604	9.806811	9.80699807	0.01611804
ρ /kgm^{-3}/	Density	"Density.dis" 	850	999.969	892.51178	913.256552	49.21499315

4. Probabilistic Outputs

From the given stochastic inputs in Tab. 1, the stochastic outputs ω, Hp, H_rot and p_A were obtained using software Anthill, as shown in Tab. 2. All calculations are performed and evaluated for 5×10^6 pseudo-random simulations using the Monte Carlo method.

Tab. 2: Stochastic outputs and their basic characteristics.

Stochastic outputs	Description	Min.	Max	Median	Mean	St. deviation
/$rads^{-1}$/	Angular velocity 13.8231 ± 2.7646 	11.0584	16.5876	13.8258	13.8231	0.908848
H_p /m/	Rotational paraboloid height 0.02446 ± 0.010045 	0.01540	0.03549	0.02435	0.02446	0.003201
H_rot /m/	Total height of water at the wall caused by rotation 0.09223 ± 0.00883 	0.08383	0.10149	0.09220	0.09223	0.002074
p_A /Pa/	Tlak v místě A, viz Fig. 1(b) $895.627^{+185.973}_{-146.599}$ 	749.028	1081.60	886.295	895.627	56.5633

The maximum calculated hydrostatic pressure is in the interval from 794.02 to 1081.60 Pa, see Table 2.

The FS reliability function for container overflow was then defined as

$$FS = H - H\_rot. \tag{3}$$

The reliability function and the 2D histogram of H versus H_rot are shown in Fig. 2. From the figures and equation (1), it can be seen that when FS>0, the magnitude of the water surface height near the container wall caused by rotation is smaller than the height of the container wall, i.e., no overflow of the container occurs. Otherwise, if FS≤0, overflow occurs.

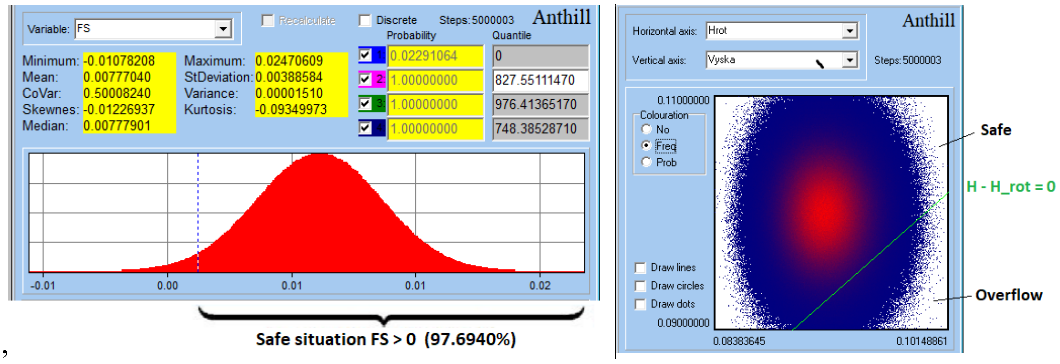


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Fig. 2: Probabilistic reliability assessment (SBRA Method, Anthill 2.6 software).

The probability of overfilling of a rotating liquid container is 2.306% as can be seen in Figure 2. In 97.694%, overfilling of the container does not occur.

5. Conclusions

A probabilistic (stochastic) approach has been applied to the problem of a rotating container with liquid. The six input random variables were expressed by bounded histograms with normal probability distribution. The overflow probability was obtained by analysing the FS reliability function using a Monte Carlo simulation technique. The result varies each time in pseudo-random repetition, so it is important to choose a sufficient number of simulation steps. In this case, 5×10^6 simulations were performed. The probability of overflow for the given input parameters is 2.306% and the maximum hydrostatic pressure ranges from 794.02 to 1081.60 Pa. The obtained knowledge can be used in designing rotating vessel or evaluating the reliability of process using rotating fluids. The application of the probabilistic approach is novel in the solution of this problem.

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