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USE OF ADVANCED KINEMATIC HARDENING RULE WITHIN FE RATCHETING SIMULATION

Klepač V.*, Parma S.*, Svárovský J., Marek R.*, Plešek J.***

Abstract: *Ratcheting still remains as a critical process for simulations. To capture material behavior, there is a vast of models of plasticity, from which we select rules of kinematic hardening (KH). To present whole workflow of ratcheting predicition, the model of Multicomponent Armstrong Frederick with Threshold with r modification (MAFTr) was selected. This model is based on concept of multicomponent backstress and was presented by Dafalias and Feigenbaum in 2010. We implemented the MAFTr model into commercial FE code Abaqus Standart via the UMAT interface. Implementation was verified by comparision of backstress components evolution in FE implementation and closed form solution derived for the case of uniaxial loading. To demonstrate the performance of UMAT subroutines, numerical examples and their computational demands will be shown. Even the well designed model is strongly influenced by identification of parameters. As the initial estimation, parameters will be found on the case of uniaxial monotonic loading using the closed form solution. For the case of calibration on more complex problems, the use of FE solution is required, so the computational demands are. Hence, effective computational approaches must be employed. The approach to calibration procedure will also be presented.*

Keywords: **Ratcheting, Plasticity, FEM, Implementation, Calibration.**

1. Introduction

Acumulation of plastic deformation by multiaxial ratcheting (MR) is possible cause of problems in wide range of engineering problems. On the other hand, it is still critical process for simulations by e.g. finite element (FE) method. Numerical simulations must be equipped with a mathematical model that reflects permanent changes in material caused by plasticity. Welling et al. (2017) summarizes and compares phenomenologicaly the effect of combining of different models of a yield surface distortion and kinematic hardening (KH). Within this work, however, we are focused strictly on KH rules, especially on *Multicomponent Armstrong-Frederick with Threshold with r modification* (*MAFTr*) firstly presented in (Dafalias and Feigenbaum, 2011). This model is based on concept of multicomponent backstress. For purpose of MR prediction on more complex parts, the model is implemented via UMAT subroutines coded in FORTRAN programming language. A comprehensive study of phenomenon that occur during the loading in plasticity regime can be found in (Marek et al., 2022). Other models and their implementations in different codes could be found in literature. Study of its effect on ratcheting simulation is, e.g., in (Halama, 2008).

Complex models require calibration of their parameters what leads to computational demands. Parma et al. (2018) presents a closed form solution for the model with kinematic hardening, which allows rapid computation of target-function values within the calibration procedure for the case of uniaxial loading.

The calibration of model must be based on very precise experimental data. To fully understand mechanism of the ratcheting occurence, behavior of the yield surface must be traced. Thus, methodology for the yield surface tracing is highly required. An overview of experimental program is provided in (Štefan et al., 2021).

* Institute of Thermomechanics of Czech Academy of Sciences; Dolejškova 5; 180 00; Praha; CZ, klepac@it.cas.cz
** Faculty of Mechanical Engineering, CTU in Prague; Technická 4; 160 00; Praha; CZ, jiri.svarovsky@fs.cvut.cz

2. MAFTr Model

The model is designed with assumptions of small strains, where the total strain comes as

$$\boldsymbol{\varepsilon}^{\text{tot}} = \boldsymbol{\varepsilon}^{\text{el}} + \boldsymbol{\varepsilon}^{\text{pl}}, \tag{1}$$

where $\boldsymbol{\varepsilon}^{\text{tot}}$ is the total strain tensor and $\boldsymbol{\varepsilon}^{\text{el}}$ and $\boldsymbol{\varepsilon}^{\text{pl}}$ are its elastic and plastic part, respectively. Elastic behavior of material follows Hooke's law $\boldsymbol{\sigma} = \lambda \mathbf{I} \text{tr} \boldsymbol{\varepsilon}^{\text{el}} + 2\mu \boldsymbol{\varepsilon}^{\text{el}}$, where $\boldsymbol{\sigma}$ is a stress tensor, λ and μ are Lamé's constants. $\mathbf{I}$ is the second-order unit tensor. However, to keep the notation simple, further in the text, the Hooke's law is used in general form $\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon}^{\text{el}}$, where $\mathbf{C}$ is the fourth-order stiffness tensor. Since only a phenomenon of kinematic hardening is adopted here, classical von Mises yield condition is took into account as

$$f(\boldsymbol{\sigma}, \boldsymbol{\alpha}) = \frac{3}{2} (\boldsymbol{s} - \boldsymbol{\alpha}) : (\boldsymbol{s} - \boldsymbol{\alpha}) - k^2 = 0. \tag{2}$$

where $\boldsymbol{s}$ is the deviatoric stress tensor, $\boldsymbol{\alpha}$ is the backstress tensor, and k is a fixed size of the yield surface and has value of the yield strength in pure shear. Hence, $\dot{k} = 0$. The backstress tensor is given by $\sum \boldsymbol{\alpha}_i$, where $\boldsymbol{\alpha}_i$ are backstress components. The associative flowrule is adopted here as

$$\dot{\boldsymbol{\varepsilon}}^{\text{pl}} = \lambda \frac{\partial f}{\partial \boldsymbol{\sigma}}, \tag{3}$$

where λ is the plastic multiplier. The *MAFTr* KH rule read

$$\dot{\boldsymbol{\alpha}}_i = \lambda \sqrt{\frac{2}{3}} c_i \left[\sqrt{\frac{2}{3}} a_i^s \boldsymbol{n} - \left[r \boldsymbol{\alpha}_i + (1 - r) (\boldsymbol{\alpha}_i : \boldsymbol{n}) \boldsymbol{n} \right] \right], i = 1, 2, 3 \tag{4a}$$

$$\dot{\boldsymbol{\alpha}}_4 = \lambda \sqrt{\frac{2}{3}} c_4 \left[\sqrt{\frac{2}{3}} a_4^s \boldsymbol{n} - \left[r \boldsymbol{\alpha}_4 + (1 - r) (\boldsymbol{\alpha}_4 : \boldsymbol{n}) \boldsymbol{n} \right] \left\langle 1 - \frac{\bar{a}}{f(\boldsymbol{\alpha}_4)} \right\rangle \right], \tag{4b}$$

where c_i is the backstress rate parameter and a_i^s affects a backstress saturation limit. $\boldsymbol{n}$ is unit norm of the yield surface and $\bar{a}$ is a threshold limit. Parameter r_i is a weighting factor and evolves according to

$$r_i = \frac{\sqrt{\frac{3}{2} \boldsymbol{\alpha}_i : \boldsymbol{\alpha}_i}}{a_i^s}, \tag{5}$$

where index i does not subject to summation. For the case of monotonic uniaxial loading, the backstress components might be analyticaly integrated and expressed in the closed form as

$$\alpha_i = \sqrt{\frac{2}{3}} a_i^s - \left(\sqrt{\frac{2}{3}} a_i^s - \boldsymbol{\alpha}_i^0 \right) \exp \left[-c_i \left(\varepsilon_{cum} - \varepsilon_{cum}^0 \right) \right] \tag{6a}$$

$$\alpha_i = \sqrt{\frac{2}{3}} (a_4^s \pm \bar{a}) - \left(\sqrt{\frac{2}{3}} (a_4^s \pm \bar{a}) - \boldsymbol{\alpha}_i^0 \right) \exp \left[-c_i \left(\varepsilon_{cum} - \varepsilon_{cum}^0 \right) \right], \tag{6b}$$

where ε_{cum} is the cumulative plastic strain. Description of *MAFTr* model in more detail is found in (Dafalias and Feigenbaum, 2011).

Tab. 1: Set of parameters calibrated on experimental data on SS 304, see (Hassan and Kyriakides, 1994).

	strain controled	stress controled
E [MPa]	198703.843	196858.3177
k [MPa]	120.65	120.65
$\bar{a}$ [−]	33.4835	33.4835
c_1 [−]	3000.0	3000.0
c_2 [−]	20.1798	20.18318
c_3 [−]	68.8705	70.3672
c_4 [−]	111.1196	107.7976
a_1^s [MPa]	56.9031	58.8224
a_2^s [MPa]	561.4938	578.2924
a_3^s [MPa]	9.6809	9.4513
a_4^s [MPa]	73.0898	70.2192

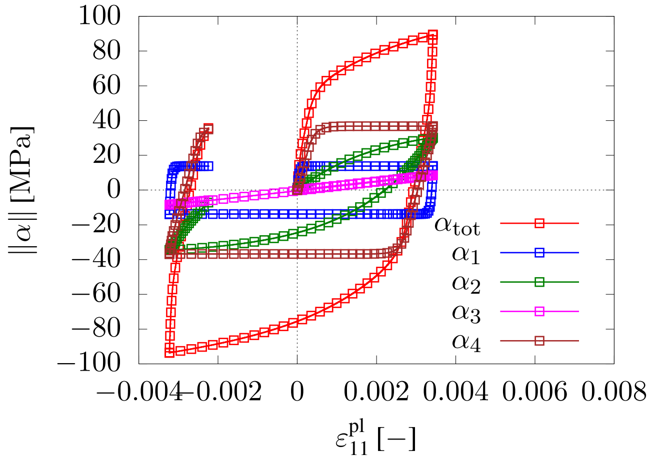

Fig. 1: Verification of implementation of MAFTr model.

3. Numerical implementation

Because of the need of simulations on complex geometries, the model was implemented into the FE code of Abaqus Standard. Following the scheme presented in (Marek et al., 2015), *MAFTr* model was coded in FORTRAN subroutines that are called from Abaqus via the UMAT interface. The algorithm is built on predictor-corrector scheme with radial return onto the yield surface. The implementation was verified by comparison with closed-form solution in Eqs. (6a) and (6b).

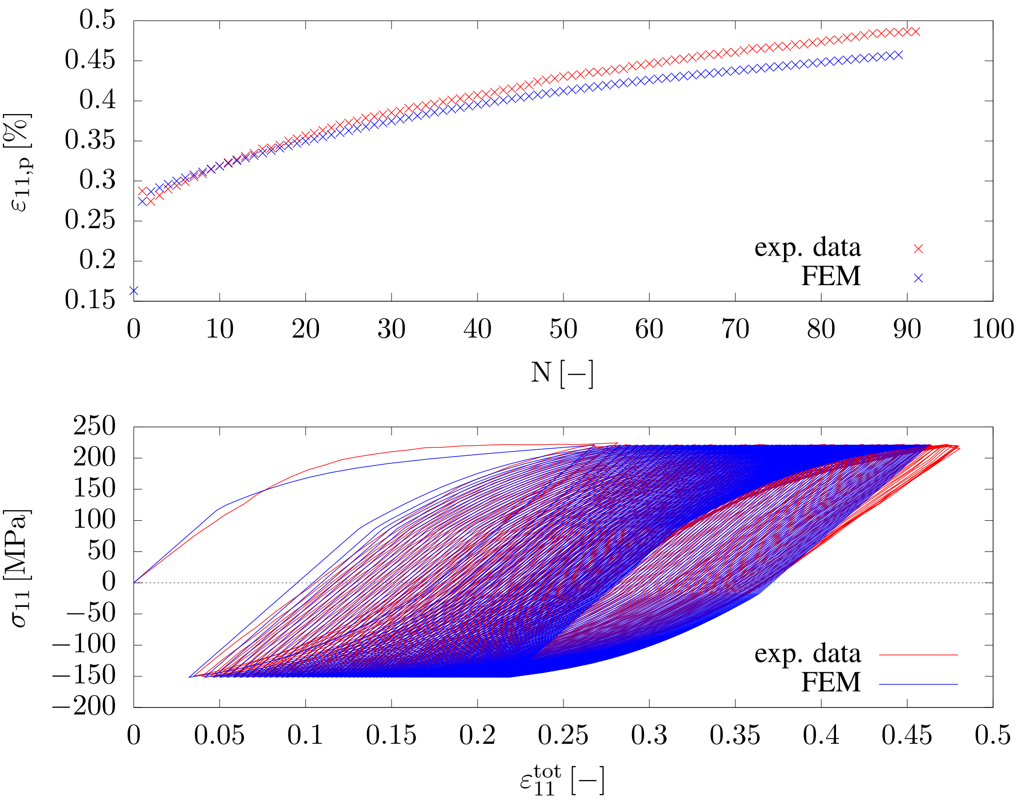

Fig. 2: Simulation of ratcheting experiments on SS 304 with mean stress $\sigma_m = 34\,\mathrm{MPa}$, and amplitude load $\sigma_a = 186\,\mathrm{MPa}$ from (Hassan and Kyriakides, 1994).

4. Model calibration

For the uniaxial case, parameters of model can be found by targeting the closed form solution (Eqs. 6a and 6b) onto the experimental data (Hassan and Kyriakides, 1994). Each run of analytical solution cost a very short computational time, hence even global algorithm can be employed.

More complicated situation comes with need of use FE method for purpose of capturing of more complex states, since FE method requires much more computational time for each iteration. To reduce the space of parameters, some constraints must be found to reduce the number of iterations. As seen above, total backstress is superposition of its components, that forms geometrical sequence. Another constraint comes from the saturation limit when $\dot{\boldsymbol{\alpha}} = 0$, hence the value of saturation parameter is

$$a_i^s = \frac{1}{\alpha_i}. \tag{7}$$

The parameters of *MAFTr* model identified for SS 304 material experimentaly treated in (Hassan and Kyriakides, 1994) are shown in Tab. 1.

5. Conclusions

The *MAFTr* model was implemented into FE computations via UMAT interface of Abaqus Standart. This implementation was verified by comparison with closed form solution, what is graphicaly demonstrated in Fig. 1. Parameters of *MAFTr* model were found by employing a series of computational methods. At the initial step, the closed-form solution was used to get values of model parameters. Subsequently, parameters were verified on the case of calibration on uniaxial and multi-axial experimental data. Comparison of prediction and experiment of uniaxial loading is plotted in Fig. 2. The simulation was driven by nominal values of ratcheting experiment. However, stress levels measured during the experiment differ from these levels in each cycle within 2% of maximum load. Thus, a difference between the prediction and experimental data exists is found due to the imperefection of the experimental data

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