27/28th International Conference
ENGINEERING MECHANICS 2022
Milovy, Czech Republic, May 9 –12, 2022
Paper **#48,** pp. 409–412, doi: 10.21495/51-2-409

ANALYSIS OF VIBRATION OF ROTORS WITH A THICK RIGID DISC ON THE OVERHANGING END SUPPORTED BY HYDRODYNAMIC BEARINGS AND LOADED BY UNCERTAIN UNBALANCE EFFECTS

Zapoměl J.[*], Ferfecki P.[**], Molčan M.[***], Kozánek J.[†]

Abstract: *Rotors of some rotating machines consist of a shaft and a rigid disc attached to its overhanging end. Because of manufacturing and assembling inaccuracies, the disc is unbalanced. In general case, the disc principal axis of inertia deviates from the shaft axis (both axes are skew lines) and the disc center of gravity is not situated on the shaft center line. Then the disc inertia effects are the source inducing the rotor lateral vibration. As magnitude of parameters specifying the disc unbalance is uncertain, application of an adequate method is required to perform the analysis. In addition, if the rotor is supported by hydrodynamic bearings, their nonlinear properties lead to rising complexity of the system vibration and to reducing its predictability. In the presented paper, the motion equation of a rigid disc has been derived. The shaft was represented by a beam body that was discretized into finite elements. The stiffness and damping parameters of the hydrodynamic bearings were linearized in vicinity of the rotor equilibrium position. Then vibration of the rotor is governed by a set of linearized motion equations. The fuzzy numbers were applied to consider uncertainty of the disc unbalance parameters. The presented procedure provides the approach to computational analysis of rotating machines with the rotor having a disc attached to its overhanging end and the geometrical, mechanical, or technological parameters of which have uncertain values.*

Keywords: Rotors, Hydrodynamic bearings, Rigid discs on overhanging end, Uncertain unbalance parameters, Fuzzy number approach.

1. Introduction

Rotors of rotating machines like centrifugal pumps, fans, turbines, or mixing devices consist of a long shaft coupled with the stationary part by bearings and of a rigid disc (impeller, wheel) placed on the overhanging end. Because of assembling and manufacturing inaccuracies, the disc is unbalanced. In general, its principal axis of inertia and the rotor axis are skew lines and the center of mass is not situated on the rotor center line. Then, the disc inertia effects become the source of the rotor excitation, which induces lateral vibration of the rotating machine. The motion equation of an unbalanced Stodola rotor with a thin disc is presented in (Gasch et al., 2002).

The disc imbalance is defined by eccentricity of the disc center of gravity and by tilting angle of the disc about the normal to the shaft center line. The values of these parameters are uncertain as the unbalance

[*] Prof. Ing. Jaroslav Zapoměl, DrSc.:
Department of Dynamics and Vibration, Institute of Thermomechanics, Dolejškova 5, Prague 8, 182 00, CZ, zapomel@it.cas.cz
Department of Applied Mechanics, VSB - Technical University of Ostrava, 17. listopadu 15, 708 33, Ostrava, CZ, jaroslav.zapomel@vsb.cz

[**] Ing. Petr Ferfecki, PhD.:
Department of Applied Mechanics, VSB - Technical University of Ostrava, 17. listopadu 15, 708 33, Ostrava, CZ, petr.ferfecki@vsb.cz
IT4Innovations National Supercomputing Center, VSB - Technical University of Ostrava, 17. listopadu 15, 708 33, Ostrava-Poruba, CZ, petr.ferfecki@vsb.cz

[***] Ing. Michal Molčan:
IT4Innovations National Supercomputing Center, VSB - Technical University of Ostrava, 17. listopadu 15, 708 33, Ostrava-Poruba, CZ, michal.molcan@vsb.cz

[†] Ing. Jan Kozánek, CSc.:
Department of Dynamics and Vibration, Institute of Thermomechanics, Dolejškova 5, Prague 8, 182 00, CZ, kozanek@it.cas.cz

results from technological operations that are always influenced by stochastic factors. Several methods have been developed for solving such problems. The approach based on utilization of fuzzy numbers assumes that the value of each uncertain quantity is not a single number but that it is a number from a certain interval and that to each value from this interval the degree of membership, the magnitude of which runs between 0 and 1, is assigned. The following solution consists in performing the interval analysis for chosen values of the membership function, which is the same for all uncertain parameters. The fuzzy number approach makes it possible to consider credibility of the obtained results. This method was applied for solving problems with uncertain parameters in different fields of mechanics: analysis of the frequency-response function of damped structures (Moens et al., 2005), vibration analysis of vehicles (Sága et al., 2005), damage prediction of mechanical structures (Vaško et al., 2013) or rotor dynamics (Zapoměl, 2021).

In the presented paper, the rotor is supported by two hydrodynamic bearings. The finite element method was employed to perform the analysis. The shaft was represented by the Euler beam body, which was discretized into finite elements (Zapoměl, 2007). The motion equations of a thick rigid disc were derived and implemented in the rotor equation of motion. The stiffness and damping parameters of the hydrodynamic bearings were linearized in neighborhood of the rotor equilibrium position. The uncertain parameters specifying the disc unbalance were expressed by fuzzy numbers. The goal was to evaluate the rotor steady state response.

2. Motion equations of the rotor with a disc attached to its overhanging end

The subject of investigation is a rotor with an overhanging end, to which a thick rigid disc is attached (Fig. 1). The rotor rotates at constant angular speed. The disc is assumed to be absolutely rigid and symmetric relative to its middle plane. The disc center of gravity is not situated on the shaft center line and the disc principal axis of inertia is slightly deviated from the shaft axis.

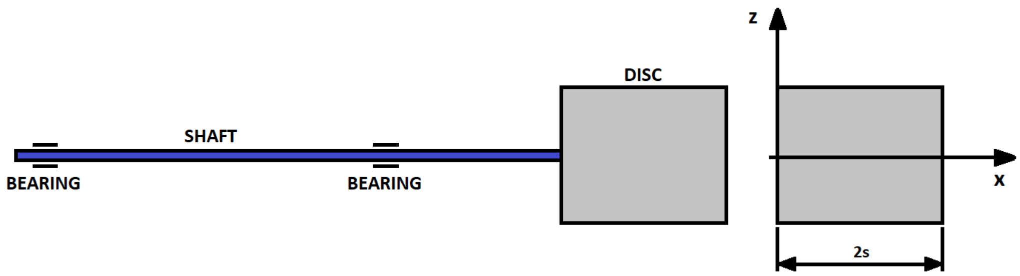

Fig. 1: Scheme of the rotor with a disc attached to its overhanging end.

The motion equations of the decoupled disc have been built up by application of the first and second impulse theorem

$$m_D \ddot{y} + m_D s \ddot{\varphi}_z = m_D e_T \omega^2 \cos(\omega t + \psi_T) + F_{Dy} - T_y \tag{1}$$

$$m_D \ddot{z} - m_D s \ddot{\varphi}_y = m_D e_T \omega^2 \sin(\omega t + \psi_T) + F_{Dz} - m_D g - T_z \tag{2}$$

$$\begin{gathered} -m_D s \ddot{z} + (J_D + m_D s^2)\, \ddot{\varphi}_y + \omega J_a \dot{\varphi}_z = \\ = (J_D - J_a)\sigma\omega^2 \cos(\omega t + \psi_S) - m_D e_T \omega^2 s \sin(\omega t + \psi_T) + m_D g\, s - F_{Dz} s + M_{Dy} - M_{oy} \end{gathered} \tag{3}$$

$$\begin{gathered} m_D s \ddot{y} + (J_D + m_D s^2)\, \ddot{\varphi}_z - \omega J_a \dot{\varphi}_y = \\ = (J_D - J_a)\sigma\omega^2 \sin(\omega t + \psi_S) + m_D e_T \omega^2 s \cos(\omega t + \psi_T) + F_{Dy} s + M_{Dz} - M_{oz} \end{gathered} \tag{4}$$

m_D is the disc mass, J_D, J_a are the disc diametral and axial moments of inertia, F_{Dy}, F_{Dz}, M_{Dy}, M_{Dz} are the forces and moments acting on the disc in the y and z directions (point of action is the disc center), T_y, T_z, M_{oy}, M_{oz} are the shear forces and bending moments, by which the disc acts on the shaft at location of its attachment, s is the half length of the disc, ω speed of the rotor rotation, g is gravity acceleration, y, z are displacements of the point on the shaft axis, at which the disc is attached to the rotor, in the y, z directions, φ_y, φ_z are angles of rotation on the shaft cross section, by which the disc is connected to the rotor, about axes y, z, t is the time, e_T is eccentricity of the disc center of gravity, σ is the tilting angle of the plane of the disc principal axes of inertia (diametral) relative to the plane perpendicular to the shaft axis, ψ_T, ψ_s are the phase shifts, and $(\dot{\ })$, $(\ddot{\ })$ are the first and second derivatives with respect to time.

The shaft is represented by a beam body that is discretized into finite elements. The hydrodynamic bearings are represented by force couplings. The oil pressure distribution in the bearing gap is calculated by solving the Reynolds equation. After implementing the disc motion equations, the equation of motion of the whole rotor system takes the form

$$\mathbf{M}\,\ddot{\mathbf{x}} + (\mathbf{B}_P + \mathbf{B}_M - \omega\mathbf{G})\,\dot{\mathbf{x}} + (\mathbf{K} + \omega\mathbf{K}_C)\,\mathbf{x} = \mathbf{f}_A + \mathbf{f}_H \tag{5}$$

$\mathbf{M}$, $\mathbf{G}$, $\mathbf{K}$, $\mathbf{K}_C$ are the mass, gyroscopic, stiffness, and circulation matrices, $\mathbf{B}_P$, $\mathbf{B}_M$ are the matrices of external and the shaft material damping, $\mathbf{f}_A$, $\mathbf{f}_H$ are the vectors of applied and hydraulic forces and $\mathbf{x}$ is the vector of the rotor generalized displacements.

Consequently, the vector of hydraulic forces was linearized in neighborhood of the rotor equilibrium position. After this manipulation the motion equation of rotor system reads

$$\mathbf{M}\,\ddot{\mathbf{x}} + (\mathbf{B}_P + \mathbf{B}_M - \omega\mathbf{G} - \mathbf{D}_B)\,\dot{\mathbf{x}} + (\mathbf{K} + \omega\mathbf{K}_C - \mathbf{D}_K)\,\mathbf{x} = \mathbf{f}_A + \mathbf{f}_{HE} \tag{6}$$

$\mathbf{D}_B$, $\mathbf{D}_K$ are Jacobi matrices of partial derivatives, $\mathbf{f}_{HE}$ is the vector of hydraulic forces related to the rotor equilibrium state.

3. Fuzzy analysis of the rotor steady state vibration

The analyzed rotor consists of a shaft and one disc attached to its overhanging end and is supported by two hydrodynamic bearings (Fig. 2). The rotor is made of steel. It rotates at constant angular speed 300 rad/s, is loaded by its weight, and excited by the disc unbalance. The mass of the rotor is 815 kg, its total length 2 500 mm, the length of the span between the bearings 1 200 mm, and the length of the disc is 500 mm. The maximum rotor diameter is 250 mm. The rotor is loaded by its weight and by the stationary force of 10 kN acting on the disc in the vertical direction.

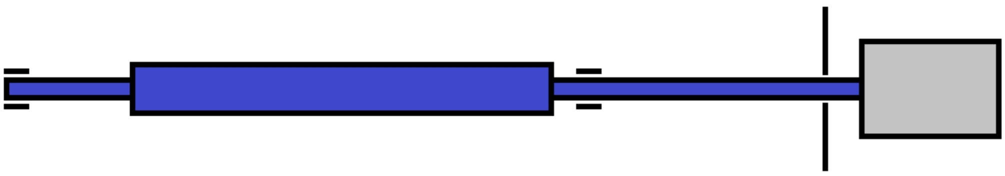

Fig. 2: Scheme of the analyzed rotor.

The disc unbalance is defined by eccentricity of the disc center of gravity, the angle of the disc tilting about the normal to the shaft center line and by the tilting phase shift (angle formed by the line going through the disc center and the center of gravity and the axis of tilting). Magnitudes of these parameters are uncertain, therefore, they are quantified by the fuzzy numbers. Their values and the corresponding membership functions are drawn in Fig. 3.

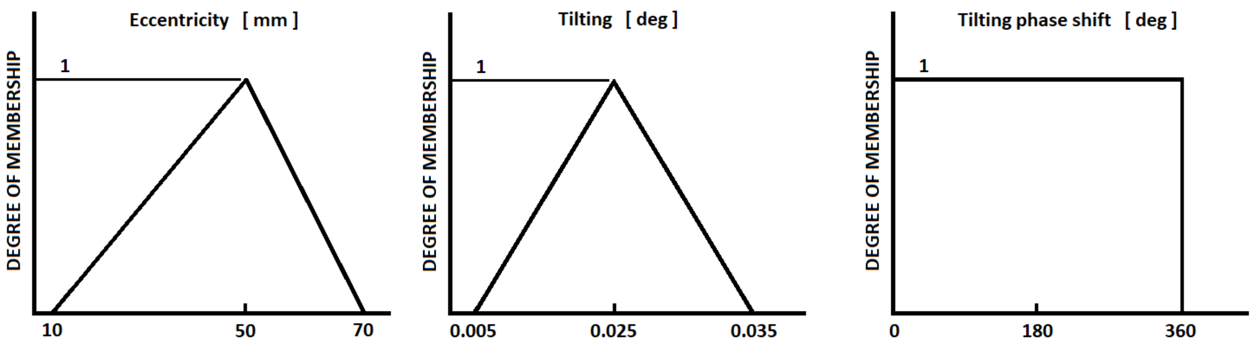

Fig. 3: The disc uncertain unbalance parameters.

The task was to determine maximum radial displacement of the shaft center at the specified location near to the disc. The results are summarized in Fig. 4.

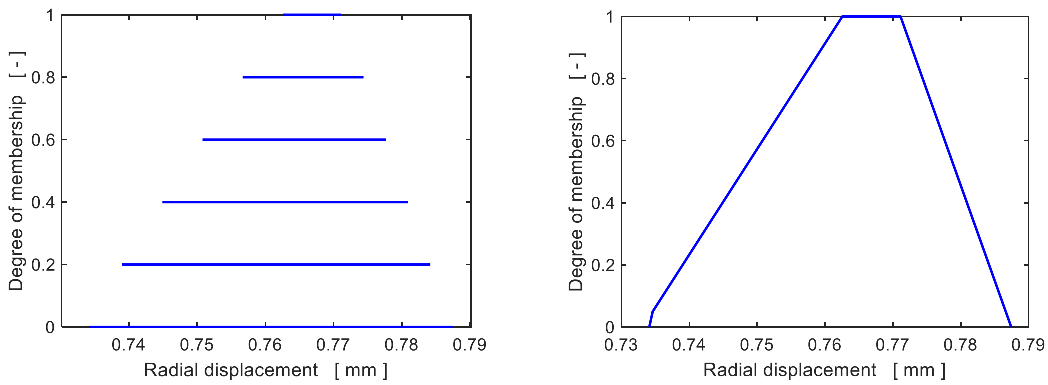

Fig. 4: The shaft maximum radial displacement expressed by the fuzzy number.

Table 1 presents the degree of credibility of the individual intervals of the shaft radial displacement at the specified location.

Tab. 1: Credibility of the radial displacement.

Radial displacement	Degree of credibility
734 – 787 μm	100.0 %
739 – 784 μm	84.7 %
745 – 781 μm	67.5 %
751 – 778 μm	50.4 %
757 – 774 μm	33.2 %
763 – 771 μm	16.1 %

4. Conclusions

The developed procedure provides the approach to analysis of a wide class of rotating machines, the rotor of which has a rigid disc attached to its overhanging end and is loaded by force effects of uncertain magnitude. The solution is based on application of the fuzzy numbers. The advantage is that the procedure does not require both knowledge of the probability density function of uncertain parameters nor corresponding random number generators. Construction of the membership function requires experience of technicians, engineers or other investigators who solve the problem and adequate knowledge database.

Acknowledgement

The support of the research work by projects RVO: 61388998, LQ1602 (NPU II) is highly acknowledged.

References

Gasch, R., Nordmann, R. and Pfützner, H. (2002) *Rotordynamics*. Springer, Berlin, Heidelberg (in German).

Moens, D. and Vandepitte, D. (2005) A fuzzy finite element procedure for the calculation of uncertain frequency-response function of damped structures: Part 1 – Procedure. *Journal of Sound and Vibration*, 288, pp. 431-462

Sága, M., Letrich, M. and Kocúr R. (2005) An analysis of vehicle vibration with uncertain system parameters. *Komunikácie*, 7(1), pp. 16-21

Vaško, M. and Sága, M., (2013) Application of Fuzzy Structural Analysis for Damage Prediction Considering Uncertain S/N Curve. *Applied Mechanics and Materials*, 4(20), pp. 21-29.

Zapoměl, J. (2007) *Computer Modelling of Lateral Vibration of Rotors Supported by Hydrodynamical Bearings and Squeeze Film Damper*. VSB-Technical University of Ostrava, Ostrava (in Czech).

Zapoměl J., Ferfecki P. and Molčan M. (2021) The fuzzy approach for investigation of the steady-state response of rotors excited by loading effects of uncertain magnitude, In Adámek, V., et al., eds, *Proceedings of Computational Mechanics* 2021, Srní, pp. 274-277.