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# MULTI-PARAMETER MTS CRITERION USED FOR ESTIMATION OF THE CRACK DEFLECTION ANGLE NEAR A CORROSION PIT

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**Abstract:** *Combined fatigue and corrosion damage is very typical for metal materials subjected to aggressive environment. Unfortunately, this topic is still not described sufficiently. Thus, a high strength steel specimen was subjected to a numerical analysis studying the mutual interaction of a fatigue crack and a corrosion pit within this work. Finite element method was utilized to model an inclined surface crack near a corrosion pit and then, the multi-parameter form of maximum tangential stress criterion was applied to estimate the crack deflection angle. Because the simulations were linear and parametric, it was easy to investigate effect of individual parameters on the results obtained. In the paper presented, the influence of the crack length, the number of the Williams expansion terms considered for tangential stress approximation and the critical distance, where the fracture criterion was applied, is discussed. The conclusions formulated based on the results of the study shall help to better understand how the fatigue crack behavior can be affected via presence of corrosion defects.*

**Keywords: HSS steel, Fatigue, Corrosion pit, Multi-parameter MTS criterion, Finite element method.**

## 1. Introduction

There exists a need to search for novel materials based on specific requirements on their properties. High-strength steels (HSS) are one of them in civil engineering applications because of their excellent strength/weight ratio. The most significant examples of their use in practice are introduced in the book of International Association for Bridge and Structural Engineering (2005). Basic mechanical properties for selected kinds of HSS can be found for instance in Toubal et al. (2020) or Wang et al. (2020). HSS are more and more often utilized in load-bearing structures where are exposed to various types of loading and/or also to different environment including an aggressive one. This leads to most typical damage phenomena of HSS: fatigue and/or corrosion. Here are introduced at least several recent works dealing with combination of both damage mechanisms: Brennan (2014), DuQuesnay et al. (2003), Jiang et al. (2009), Jiang et al. (2018) or Wang et al. (2014).

In this work, deflection of a short inclined fatigue crack near a corrosion pit is investigated via multi-parameter form of the Maximum Tangential Stress (MTS) criterion. This approach is based on using approximation of the tangential stress by means of Williams series expansion (see Williams, 1957). Author of this work published several works on application of the criterion mentioned on various materials, see e.g. Malíková (2015).

## 2. Methods and problem description

Fig. 1 shows a schema of the rectangular specimen with a short inclined fatigue crack and a nearby corrosion pit loaded via remote uniaxial tensile stress range assumed within the analysis. The values of the individual parameters considered are as follows: the distance between the crack and the edge of the corrosion pit *const* = 0.1 mm, the corrosion pit length 2*P* = 4 mm, the corrosion depth *D* = 1 mm, the total

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specimen length $L$ = 100 mm, the specimen width $W$ = 10 mm, the crack length varied between 0.05 and 4 mm ($a$ = 0.05, 0.25, 0.5, 1, 1.5, 2, 3 and 4 mm), the crack inclination angle varied between -45° and +45° ($\gamma$ = -45, -30, -15, 0 15, 30 and 45°) and the applied tensile stress range $\Delta\sigma$ = 300 MPa.

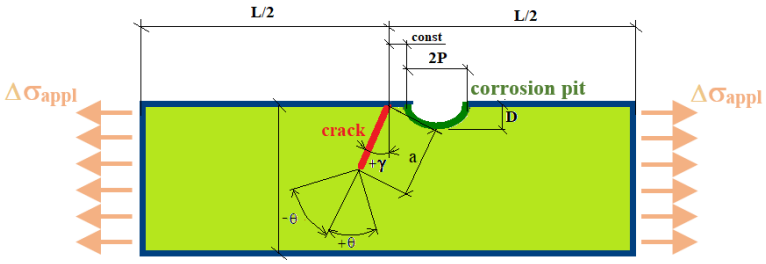


*Fig. 1: Schema of the rectangular specimen with a short inclined fatigue crack and a corrosion pit loaded via remote uniaxial tensile stress range.*

The model was created in the commercial finite element system ANSYS considering plane strain conditions and using 8-nodes quadrilateral PLANE183 elements with refined mesh around the crack tip (the element size at the crack tip was about 0.001 mm). Properties of the linear elastic material model assumed were set to be 210 GPa for the Young's modulus and 0.3 for the Poisson's ratio (as it is typical for high strength steels).

The results of the numerical simulations were then applied as inputs for so-called over-deterministic method (see e.g. Ayatollahi and Nejati, 2011) serving for estimation of the higher-order terms of the Williams expansion (WE) for approximation of crack-tip stress field. Then, the stress tensor components can be expressed in such a form:

$$\sigma_{ij} = \sum_{n=1}^{N} \frac{n}{2}\, r^{\frac{n}{2}-1} A_n\, f_{\sigma_{ij}}(\theta, n) + \sum_{m=1}^{M} \frac{m}{2}\, r^{\frac{m}{2}-1} B_m\, g_{\sigma_{ij}}(\theta, m) \quad \text{where} \quad i, j \in \{x, y\}. \tag{1}$$

The meaning of the symbols in Eq. 1 is as follows: $\sigma_{ij}$ are stress tensor components, $A_n$ and $B_m$ are coefficients of the WE corresponding to loading mode I and II, respectively, $N$ and $M$ are number of the WE terms considered for loading mode I and II, respectively, $(r, \theta)$ are coordinates in the polar coordinate system with its origin at the crack tip and $f_\sigma$ and $g_\sigma$ are known dimensionless functions related to loading mode I and II, respectively.

Similarly to Eq. 1, an equation can be written also for displacement vector components and it is then directly utilized as the basis of the ODM method. Displacements of a selected set of nodes together with their coordinates enable calculation of the coefficients $A_n$ and $B_m$. Particularly, the nodes at the radius of 0.01 mm from the crack tip were considered for ODM application within this paper and 19 initial WE terms were calculated for each configuration. It is always better to apply the ODM at larger distances from the crack tip, but it was not possible in this case because of the closeness of the corrosion pit and the crack tip. This can be the reason, why the generalized MTS criterion does not work properly when considering more higher-order terms of the WE.

When the coefficients $A_n$ and $B_m$ are known, the tangential stress can be expressed via the WE and following the main idea of the MTS criterion (see Erdogan and Sih, 1963), the direction of further crack propagation can be found. The condition of the MTS criterion can be expressed mathematically like:

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0 \quad , \quad \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} < 0 \quad . \tag{2}$$

Eq. 2 represents the condition of the maximum of the function $\sigma_{\theta\theta}$ which is the tangential stress. It should be mentioned that in contrast to the classical one-parameter MTS criterion, the generalized multi-parameter MTS criterion needs to be applied at a particular radial distance. This distance is often called *critical distance* and should be related to the material properties. Its value used for metals is of order of several tenths of millimetres. Therefore, $r_c$ = 0.1 and 0.5 mm was considered within the study presented.

## 3. Results

As stated above, the goal of this analysis was to assess the mutual interaction between an inclined fatigue crack and a corrosion pit. Particularly, the angle of further crack propagation was studied via multi-

parameter MTS criterion for various crack lengths ($a$ = 0.05 to 4 mm), various critical distances ($r_c$ = 0.1 or 0.5 mm) and various numbers of the WE terms (NM = 1, 2, 3 and 4) considered in the tangential stress approximation. The results obtained for selected initial crack inclination angles $\gamma$ = -30° and +30° can be seen in Fig. 2. For comparison, also the results for a specimen without the corrosion pit are presented.

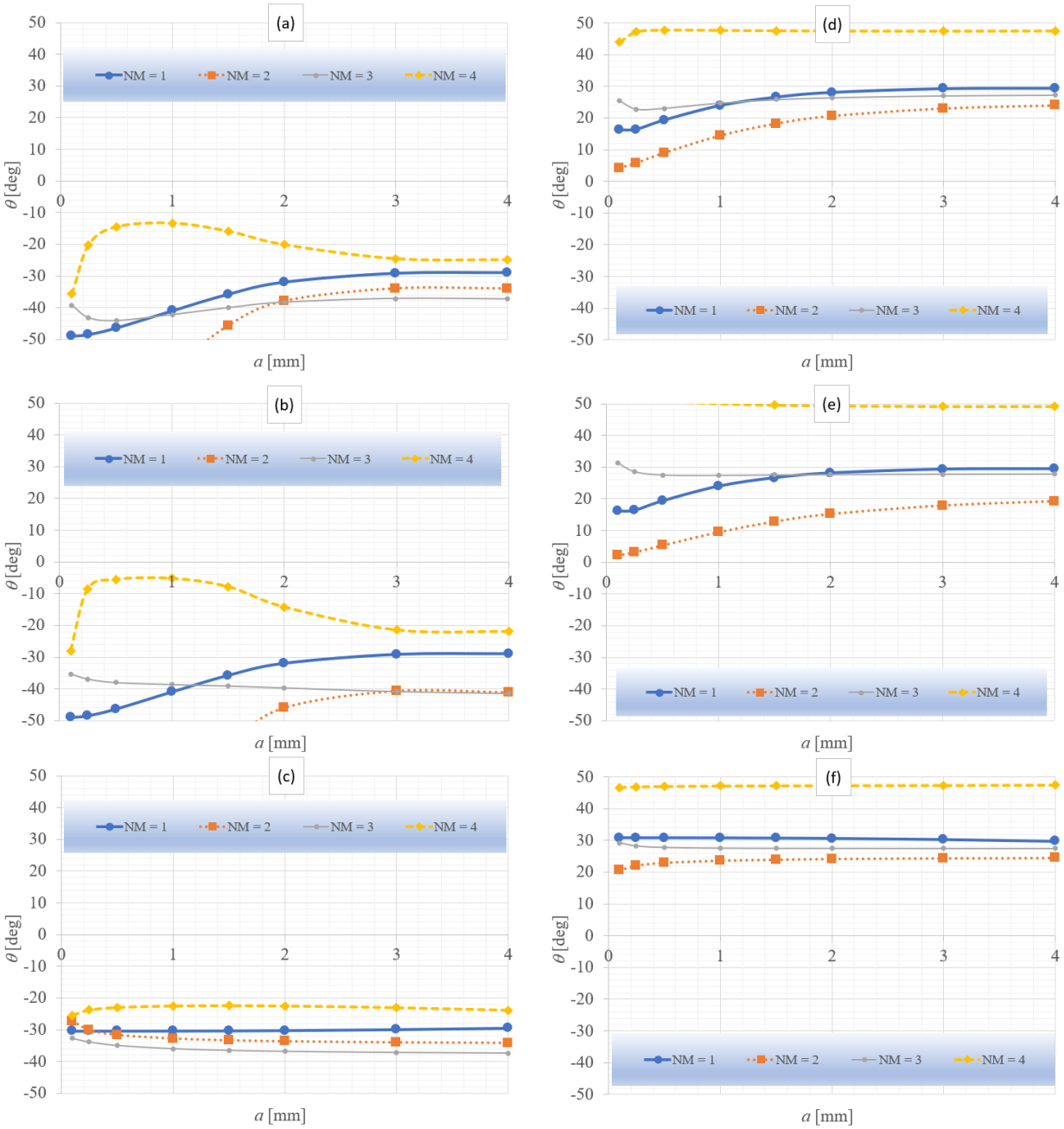


*Fig. 2:Dependence of the further crack propagation direction on the crack length for various numbers of the WE terms: left column $\gamma$ = -30° with (a) $r_c$ = 0.1 mm, (b) $r_c$ = 0.5 mm, (c) without the corrosion pit; right column $\gamma$ = 30° with (d) $r_c$ = 0.1 mm, (e) $r_c$ = 0.5 mm, (f) without the corrosion pit.*

From the results presented in Fig. 2, several points can be stated:

- Results estimated via multi-parameter MTS criterion at both critical distances seems to be rather similar.
- Considering that a crack very often tries to propagate perpendicularly to the opening stress, the crack deflection angle for the specimen without the corrosion pit should reach values about -30° (Fig. 2 (c)) and 30° (Fig. 2 (f)). Differences observed especially for short cracks in Fig. 2 (a) and (b) and Fig. 2 (d) and (e) are thus related to the mutual interaction of the crack-tip stress field and the nearby corrosion pit.
- It seems from the results that considering higher-order terms in the MTS criterion does not improve the prediction of the fatigue behavior of the configuration under the study.

- Unfortunately, it is not easy to assess unambiguously the effect of the particular number of WE terms considered for the generalized MTS criterion.

- The more inclined is the crack towards the corrosion pit (negative $\gamma$ values, i.e. left column in Fig. 2) the more unstable is the behavior of the solution via multi-parameter MTS criterion. This is assigned to overlapping of the stress fields of both the crack and the corrosion pit, which can affect also accuracy of the higher-order terms coefficients determined via ODM. Moreover, the numerical solution of Eq. (2) is poorly conditioned and automatized searching for the appropriate root for such a big amount of various configuration is thus difficult.

Finally, it shall be concluded that more numerical investigations are absolutely necessary to be able to assess the mutual interaction of an inclined fatigue crack and a corrosion pit accurately. Experimental testing could also bring data necessary for the mutual comparison of both analyses.

## 4. Conclusions

In the paper presented, a mutual interaction between an inclined fatigue crack and a corrosion pit is investigated. Particularly, the crack deflection angle is estimated via multi-parameter form of MTS criterion considering various numbers of the WE terms in the tangential stress approximation and two various critical distances. It has been found out, that the corrosion pit affects the crack behavior when the crack is rather short. Nevertheless, it has been shown that the classical one-parameter form of the MTS criterion seems to predict the crack deflection angle better than the multi-parameter one. The results obtained behaved generally in a very unstable manner because of both overlapping of the crack tip stress field and corrosion pit and poorly conditioned numerical solution of Eq. (2). More numerical and experimental studies are necessary when the mutual effect of fatigue and corrosion phenomenon in HSS (or in other materials) shall be assessed.

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