29th International Conference
ENGINEERING MECHANICS 2023
Milovy, Czech Republic, May 9 – 11, 2023

SEMI-PROBABILISTIC APPROACHES FOR ACTION EFFECTS

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Abstract: *The computational burden of Monte Carlo (MC) type simulation represents the main obstacle to the approach for time-consuming finite element (FE) computational models since it is not computationally feasible. Consideration of uncertainties, therefore, remains usually at the level of partial safety factors, the approach which is in most cases on the very conservative side. That is why alternative techniques of so-called safety formats are becoming more and more attractive recently. Methods focused on the Estimation of Coefficient of Variation (ECoV) of structural resistance have been developed and applied recently. They represent a compromise between the simple and in most cases conservative approach of partial safety factors and highly computationally demanding MC simulation. Semi-probabilistic methods have been applied for structural resistance assessment, but not for the action effects side. The paper shows the possibility of application of these methods for action effect – load side of safety margin. Selected efficient semi-probabilistic methods based on the ECoV method according to fib Model Code 2010 and an improved approach called Eigen ECoV, are utilized. The application of approaches for the assessment of concrete tunnel linings is shown.*

Keywords: Semi-probabilistic approach, Safety formats, Action effect, Concrete tunnel linings.

1. Introduction

Semi-probabilistic approaches using advanced probabilistic techniques are becoming more and more attractive recently. Generally, due to extreme computational burden of each numerical simulation in computational modelling of structures, it is not feasible to perform fully probabilistic assessment of the structure by standard Monte Carlo simulation technique. Engineers in current practice are mostly familiar with partial safety factors approach, however more advanced methods are gaining attraction due to the fact, that they allow for rationalized and more efficient engineering verifications. Semi-probabilistic methods are advantageous since they offer a balance between accuracy and efficiency. Typical representatives in this category are the standard ECoV method (Červenka, 2013), Taylor Series Expansion (Novák & Novák, 2020), Numerical Quadrature by Rosenblueth (1975), and recently proposed Eigen ECoV (Novák & Novák, 2021). These simplified methods are based on several strong assumptions which are typically valid in structural engineering, which allow estimating the mean value and variance (coefficient of variation) of the quantity of interest by simple analytical formulas and only a few simulations.

This article explores the possibilities of applying safety formats for action effect. Comparative calculations are then demonstrated using a plane-strain FE model from a realistic tunnel project in soft soil. Note that in contrast to most of the existing applications of ECoV methods focused on the resistance of structural elements (Slowik et al., 2021; Novák et al., 2018; Novák et al., 2022) this paper shows the application of ECoV methods for the descriptions of action effects obtained from the FE model, which represents a novelty of the approach.

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ISSN: 1805-8256, doi: 10.21495/em2023-175

2. Semi-probabilistic approaches

2.1. Separation of resistance and action effect

The basic reliability concept is given by the following expression, where Z(X) is the safety margin, which is defined as the difference between structural resistance *R* and action effect *E*:

$$Z(X) = R - E \tag{1}$$

The probability of the negative safety margin (probability of failure) is used in a fully probabilistic method to prove the safety requirements. The semi-probabilistic approaches assume the separation of two random variables structural resistance *R* and action effect *E* through their design values:

$$R_d = F_R^{-1}\,(-\alpha_R \beta), \text{ and} \tag{2}$$

$$E_d = F_E^{-1}\,(-\alpha_E \beta), \tag{3}$$

where F^{-1} represents inverse cumulative distribution function, α is a sensitivity factor and β is the target reliability index (both can be found in Eurocodes reflecting the type of the structure). The paper focuses on the estimation of E_d when the function of action effects *E(***X***)* of input random vector (**X** being a vector of *N* basic variables) is solved by a 2D FE model. Based on these assumptions, the probability distribution is fully described by mean value and CoV, and reliability analysis thus reduces to the estimation of the first two statistical moments. The selected ECoV methods are described below.

2.2. Standard ECoV according to fib model code

Standard ECoV method often used in analyses of concrete structural members was developed by Červenka (Červenka, 2013) and later implemented into the *fib* Model Code 2010 (fib, 2013). It is based on a simplified formula for the estimation of a characteristic value of a lognormal variable with the mean value μ_R and CoV v_R using only two numerical simulations R_m (with mean values of input variables) and R_k (using characteristic values of input variables). The formula of the standard ECoV method is:

$$v_R = \frac{1}{1.645} ln\,\left(\frac{R_m}{R_k}\right). \tag{4}$$

Note that this paper is focused on action effects E and thus we should assume Gaussian distribution instead of Lognormal distribution, thus the standard ECoV formula used in this work is as follows:

$$v_E = \frac{E_m - E_k}{1.645\, E_m}. \tag{5}$$

2.3. Eigen ECoV

The recently proposed Eigen ECoV (Novák & Novák, 2021) is based on the idea of projecting input random vector on 1D eigen distribution Θ with variance equal to the first eigenvalue of input covariance matrix $\sigma_\Theta^2 = \sum \sigma_{X_i}^2 = \lambda_1$ and mean value is simply obtained as:

$$\mu_\Theta = \sqrt{\textstyle\sum_{i=1}^{N} \left(\mu_{X_i}\right)^2}. \tag{6}$$

In the original proposal, there are three levels of Eigen ECoV. The most promising Eigen ECoV formula for the estimation of v_E offering a balance between efficiency and accuracy is:

$$v_E \approx \frac{3E_m - 4E_{\Theta\frac{\Delta}{2}} + E_{\Theta\Delta}}{\Delta_\Theta} \cdot \frac{\sqrt{\lambda_1}}{E_m}, \tag{7}$$

where simulation $E_{\Theta\Delta} = e(X_{\Theta\Delta})$ is calculated with coordinates of input realization $X_{\Theta\Delta} = (X_{1\Delta}, \dots, X_{N\Delta})$ and $E_{\Theta\frac{\Delta}{2}} = r\left(X_{\Theta\frac{\Delta}{2}}\right)$ with coordinates $E_{\Theta\frac{\Delta}{2}} = \left(X_{1\frac{\Delta}{2}}, \dots, X_{N\frac{\Delta}{2}}\right)$. The input vectors consisting of reduced values of input random variables are $X_{i\Delta} = F_i^{-1}\big(\Phi(-c)\big)$ and the intermediate coordinates are as follows:

$$X_{i\frac{\Delta}{2}} = \mu_{X_i} - \frac{\mu_{X_i} - X_{i\Delta}}{2} = \frac{\mu_{X_i} + X_{i\Delta}}{2}. \tag{8}$$

Δ_Θ represents the distance between μ_Θ and desired quantile $F_\Theta^{-1}\big(\Phi(-c)\big)$ obtained under the assumption of Gaussian distribution as $\Delta_\Theta = c \cdot \sqrt{\lambda_1}$.

3. Concrete tunnel: 2D plane-strain non-linear FE

A tunnel cross-section with a cross-sectional area of A = 50.0 m2, has been elaborated through 2D plane-strain non-linear FE analyses (Fig. 1). The model represents a typical sprayed concrete tunnel shape with different curvatures along the perimeter, located at a depth of 20 m from the surface. The unloading Young’s modulus is simulated through a proportionally hardened material (dark blue in Fig. 1) underneath the opening. The analysis assumed full-face excavation, and to simulate the 3D stress relief and arching effects due to soil deformation ahead of the excavation face, a relaxation factor (k) was applied in the soil before the soil removal and lining installation step, see e.g. Potts (2001).

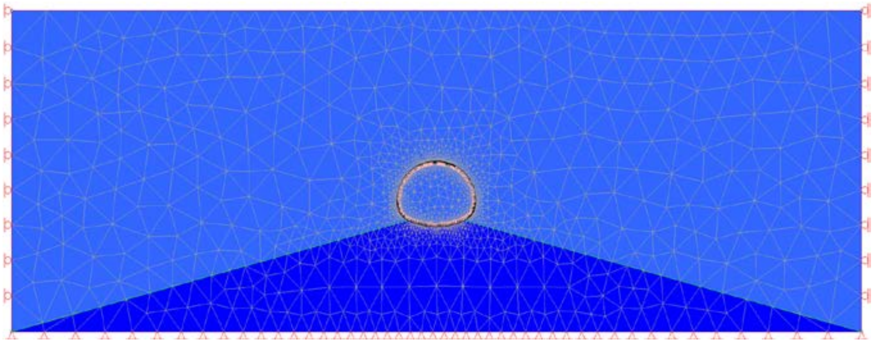

Fig. 1: Overview of the plane-strain finite element model.

An overview of the parameters used is given in Table 1. Note that all random variables are assumed to have Gaussian distribution. The geological model and the range of parameters used have been selected based on an extensive survey of available geotechnical information (see Spyridis et al. (2016)). The concrete liner was modelled with an elastic concrete material model, and it has a thickness of 300 mm, a Young‘s modulus of 13 GPa (John & Mattle, 2003) and a Poisson’s ratio of v = 0.2.

Tab. 1: Input parameters of the geotechnical analyses – stochastic model, indicating the mean values, the standard deviations and (in parenthesis) the coefficients of variation.

Soil property		**Mean value**	**Std. deviation (CoV)**
Material density, γ	[kN/m^3]	20	-
Young's modulus, E	[MPa]	200	50 (0.25)
Poisson's ratio, v	[-]	0.45	-
Undrained shear strength, s_u	[kPa]	0.20	0.05 (0.25)
Lateral stress coefficient, K_o	[-]	1.00	0.20 (0.20)
Relaxation factor, λ	[%]	67.5	6.75 (0.10)

ECoV methods were used to estimate mean value and variance in all locations of tunnel cross-section to determine local mean values (subscript μ) of moments M and axial forces N together with estimated design quantiles $E_d = F_E^{-1}\,(-\alpha_E\beta)$ assuming Gaussian distribution with estimated moments, $\alpha_E = -0.7$ and $\beta = 3.8$. Since for the assessment of the tunnel structure we ultimately use an interaction diagram obtained according to Eurocode 2 (CEN, 2005), we estimate design quantiles from both tails of the distribution to find the most dangerous combination of *N* and *M* (minimum values with subscript minus (–) and maximum values with subscript plus (+)). Obtained results can be found in Fig. 2 (*M* – top, *N* – bottom), mean values (solid blue), maximum (red) and minimum (green) values are obtained from design quantiles of Gaussian distribution. The last columns clearly show differences in ECoV methods: the estimated PDFs are shown together with design values in the selected important location for *N* and *M*. Note that obtained results for each of the methods are based on a different number of simulations: Standard ECoV - 2, Eigen ECoV - 3 (one additional simulation to standard ECoV).

4. Conclusions

This paper presents an application of existing semi-probabilistic methods for the estimation of the coefficient of variation and corresponding design values of action of load. Such application of ECoV methods for the statistical descriptions of action effects obtained from the FE model represents a novelty of the approach. Feasibility of the approach is demonstrated on a realistic tunnel structure.

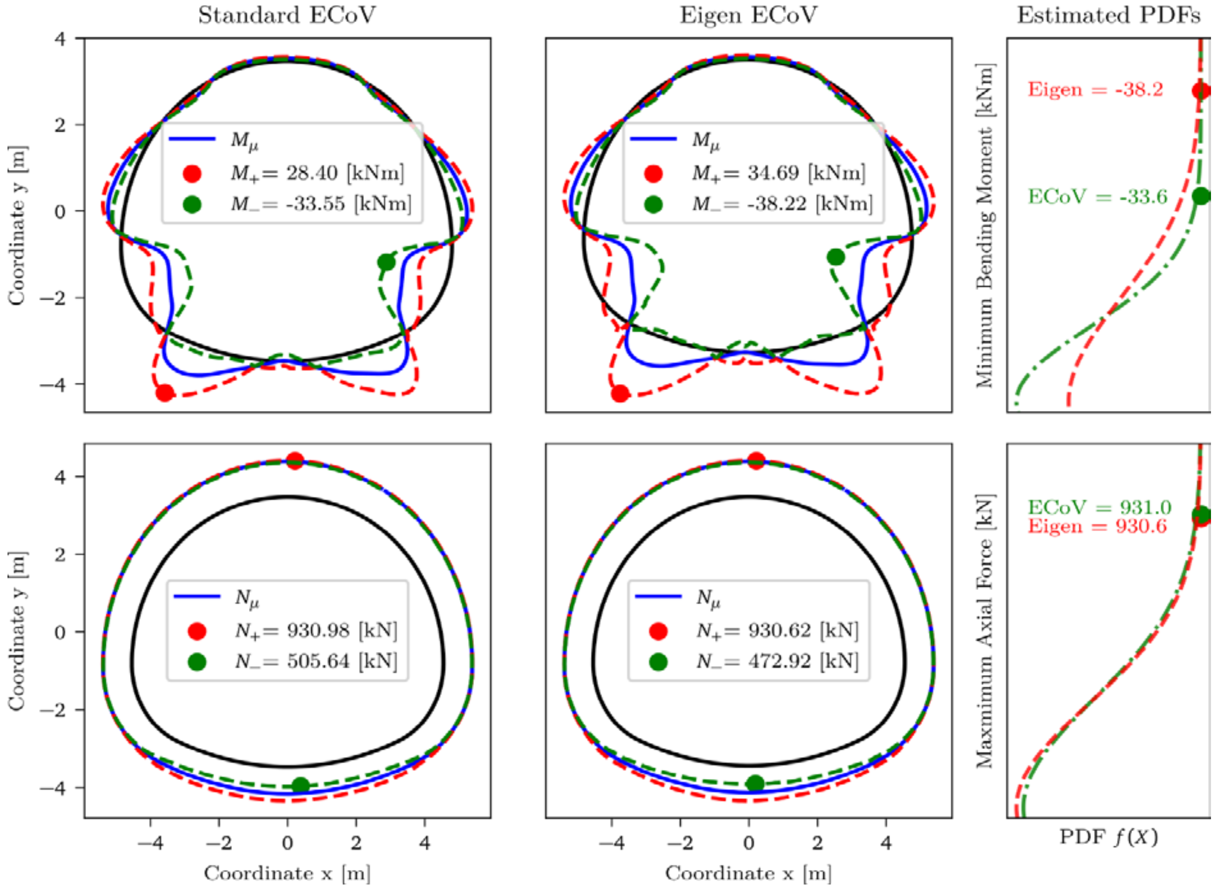

Fig. 2: Spatial distributions of axial forces F (top) and moments M (bottom) for Standard ECoV (left) and Eigen ECoV (middle). The estimated PDFs in selected locations (right).

Acknowledgement

The authors acknowledge the support from the Czech Science Foundation, project No. 22-00774S.

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