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COMPUTER SIMULATION STUDY OF THE STABILITY OF UNDERACTUATED BIPEDAL ROBOT MODELS (motivated by Griffin and Grizzle, 2017)

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Abstract: *A key feature for bipedal walkers (robots and humans as well) is their stability or disturbance rejection defined as the ability to deal with unexpected disturbances. The paper by Griffin and Grizzle (2017) have significantly contributed to the shift from flat ground to slopes and steps when evaluating the walking efficiency of their robots. Similarly, in this contribution, based on the appropriate model of robot dynamics and control law, we examine the stability of walking-without-falling for different ground perturbations for a three-link compass gait walker. I.e., we perform the sensitivity analysis of the walking stability of underactuated bipedal walker with respect to certain disturbation using the alaska/MultibodyDynamics simulation tool.*

Keywords: Mechatronics, Bipedal robot, Multibody dynamics, Acrobot, Control applications.

1. Introduction

Bipedal underactuated robots with an upper body (so-called torso) form a subclass of legged robots. The simplest underactuated walking robot hypothetically able to walk is the so-called Compass gait bipedal walker, alternatively called the Acrobot, see Fig. 1 (a). Underactuated mechanical systems, i.e. systems with fewer actuators than degrees of freedom, encounter many applications in different fields, e.g., robotics, aeronautical and spatial systems, marine and underwater systems, and in-flexible and mobile systems (Krafes et al., 2018).

The key feature of bipedal walkers (robots and humans as well) is their stability or more precisely *disturbance rejection* defined as the ability to deal with unexpected disturbances (Hobbelen and Wisse, 2007). This feature can be applied to both the powered walkers as well as passive dynamic walkers. In early 1990, McGeer (1990) demonstrated that a passive dynamic walker could walk down a shallow slope with neither control nor actuation. Then, the concept of Passive Dynamic Walking (PDW) has been used as a starting point for designing powered walkers on level ground. Although there exist several ways to measure the disturbance rejection for a PDW-based walker theoretically, e.g., Floquet multipliers, Basin of Attraction, the Gait Sensitivity Norm, see Hobbelen and Wisse (2007) and references therein; further in this study, we focus on "experimental measures", i.e., on a computer simulation study of the case when the ground step perturbation or a change in terrain slope, may cause a walker to fall. More specifically, a parameter study (e.g. having the slope as one of the parameters) of walking without falling will be carried out using the software package *alaska/MultibodyDynamics* (Alaska, 2021). I.e., inspired by the paper Griffin and Grizzle (2017), we extend our previous works Polach et at. (2022) and Papáček et al. (2022) by studying a 3D bipedal robot model implemented to *alaska/MultibodyDynamics* to walk over sloped planes and randomly placed steps, all without a priori knowledge of the environment or external sensing.

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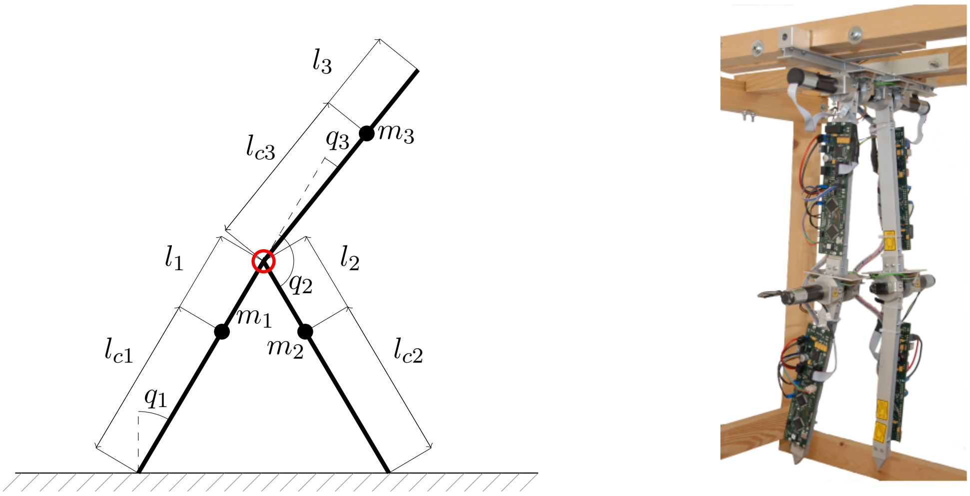
Notice that this study has been motivated by the need to implement the previously developed sensor and control algorithms for the real-time movement of the laboratory walking robot, designed and built at the Department of Control Theory of the Institute of Information Theory and Automation (ÚTIA) of the Czech Academy of Sciences, see Fig. 1 (b). A detailed description of this underactuated walking-like mechanical system (called further UTIA Walking Robot – UWR) is provided in Anderle et al. (2015) and Anderle and Čelikovský (2019).



(a) (b)

Fig. 1: (a) Compass gait bipedal walker with upper body: parameters and coordinates, (b) UTIA Walking Robot: laboratory mechatronic walking robot-like system.

2. Mathematical models of the underactuated bipedal robot

The model of the underactuated bipedal robot with an upper body, as schematically depicted in Figure 1 (a), is composed of two rigid legs (stance leg = 1, swing leg = 2) connected at their ends by a revolute joint which is equipped with an actuator. The second actuator is used to control the upper body, i.e. torso (torso = 3). In this case, the angle which is not actuated is the angle between the stance leg and the ground (denoted as q_1). The mechanical parameters of the bipedal with the torso to be considered in our simulations are taken from Čelikovský and Anderle (2021) and summed in Table 1 together with the initial conditions for angles and angular velocities.

The control approach in Čelikovský and Anderle (2021) was applied to a dynamic equation in the well-known form for mechanical systems obtained from the usual Lagrangian approach $\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{u}$, where $\mathbf{D}(\mathbf{q})$ is the inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ contains Coriolis and centrifugal terms, $\mathbf{G}(\mathbf{q})$ contains gravity terms, $\mathbf{u}$ stands for the vector of external forces, $\mathbf{q}$, and $\dot{\mathbf{q}}$ are 3-dimensional configuration vectors of angular positions and velocities, $\ddot{\mathbf{q}}$ is the vector of accelerations.

In Čelikovský and Anderle (2021), partial feedback linearization-based control of the bipedal legs resulting in the almost linearized model of bipedal legs from Čelikovský et al. (2008), i.e. $\dot{\xi}_1 = d_{11}^{-1}\xi_2, \quad \dot{\xi}_2 = \xi_3, \quad \dot{\xi}_3 = \xi_4, \quad \dot{\xi}_4 = w_2$, where d_{11} is an appropriate term of the inertia matrix and w_2 is a control input in linear coordinates was extended by virtual holonomic-based control of the torso movement, i.e. roughly saying the movement of the torso during the step is controlled such that the nonlinear term d_{11}^{-1} in the first line is linear during the step. As a result of this, the original nonlinear model of bipedal legs with the torso results in linear model $\dot{\xi}_1 = \xi_2, \quad \dot{\xi}_2 = \xi_3, \quad \dot{\xi}_3 = \xi_4, \quad \dot{\xi}_4 = w_2$ in ξ coordinates. In order to have a stable walking control of bipedal legs with the torso, the numerical optimization of mechanical parameters of the underactuated bipedal with the torso was necessary to perform using the *fmincon* solver in the MATLAB programming language such that the linear control w_2 to be applied on the original nonlinear model avoid singularities after recomputing into original $q, \dot{q}$ coordinates, Čelikovský et al. (2008). The *fmincon* solver finds one particular combination of mechanical parameters therefore any further sensitivity analysis of mechanical parameters variations is required for further implementation of the control on the UWR.

Tab. 1: Parameters of the bipedal robot with an upper body (torso) and initial conditions

Parameter	Description	Value	Unit
l_1, l_2	length of the legs	0.68	[m]
l_3	length of the torso	1.02	[m]
l_{c1}, l_{c2}	location of mass center	0.23	[m]
l_{c3}	location of mass center of the torso	1.02	[m]
m_1, m_2	mass of both legs	0.97	[kg]
m_3	mass of the torso	1.71	[kg]
I_1, I_2	moment of inertia of both legs	0.01	[kg m^2]
I_3	moment of inertia of the torso	0.60	[kg m^2]
$q_1(t_0)$	initial condition for q_1	-0.1	[rad]
$q_2(t_0)$	initial condition for q_2	3.3416	[rad]
$q_3(t_0)$	initial condition for q_3	0.1	[rad]
$\dot{q}_1(t_0)$	initial condition for $\dot{q}_1$	0.1736	[rad s^{-1}]
$\dot{q}_2(t_0)$	initial condition for $\dot{q}_2$	-0.1757	[rad s^{-1}]
$\dot{q}_3(t_0)$	initial condition for $\dot{q}_3$	-0.0876	[rad s^{-1}]

When the mathematical model of the underactuated bipedal with the torso is created in the MATLAB, then the results of the simulations look very well. This is due to the fact that the same dynamic equation representing the underactuated bipedal with the torso is used for both the development of the control approach and the simulations of its movement, as well, see Fig. 2 (a) taken from Čelikovský and Anderle (2021), where stable walking control of bipedal legs with torso even with an initial error was presented.

As the ongoing research is related to the real-time control of the laboratory walking robot shown in Fig. 1 (b), we are looking for yet another simulation tool than the MATLAB programming language. Therefore, the *alaska/MultibodyDynamics* software will be used to perform the simulation of one or multiple reference steps including the impact based on reference torques from the simulation in Čelikovský and Anderle (2021). In the 3D multibody model (i.e. not planar 2D as in Fig. 1 (a)) of the bipedal robot with the torso, created in this mechatronic software, they are, in this stage of model development, the structural parts of the bipedal modeled as rigid bodies. The 3D topological scheme of the model is similar to the topological scheme of the model created in the MATLAB programming language. The number of degrees of freedom in kinematic joints of the bipedal multibody model is 5. There is the planar joint between the ground and body 1 (stance leg), the revolute joint between body 1 and body 2 (swing leg) and the revolute joint between body 1 and body 3 (torso), see Fig. 2 (b). The torques acting in the revolute kinematic joints are prescribed and will be optimized. The contact conditions in the connection "sole" of the bipedal and the ground are modeled using the barrier functions and spring-damper elements.

Let us remark that it is sometimes more advantageous to formulate and solve the equations of motion of a studied multibody system without the usage of commercial software tools. The main reason is the limited possibility of introducing some special features and special model elements as well as some non-standard solution or optimization methods, see e.g. Hajžman and Polach (2007) and references therein.

3. Summary and outlook

Motivated by Griffin and Grizzle (2017), more precisely by the sentence: *It is hoped that other robotics researchers will consider environments other than flat ground when evaluating the walking efficiency of their robots*, this work represents the proof-of-concept study enabling to validate the previously developed sensor and control algorithms for the real-time movement of bipedal walking robots. The key role in this analysis will play the *alaska/MultibodyDynamics* simulation tool (Alaska, 2021). Resuming, the *alaska* is used for creating a multibody 3D model of the bipedal robot with an upper body to perform numerical experiments testing the ability of control algorithms of the bipedal walker to deal with unexpected disturbances. As a starting point, instead of ground step perturbation, we will experiment with the disturbance of the initial system state.

Our ongoing research concerns numerical simulations for testing the ability of various feedback methods to stabilize the robot in a certain equilibrium. Consequently, equipped by the implementation of both

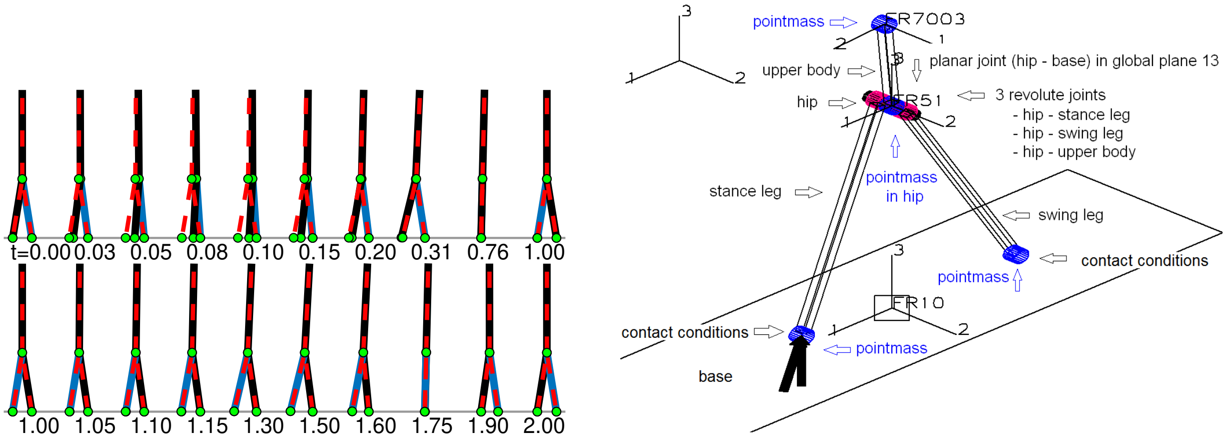


Fig. 2: (a) The animation of the underactuated biped robot with torso hybrid stable walking. The dotted line represents a reference underactuated biped with a torso to be tracked. Taken from Čelikovský and Anderle (2021). (b) Alaska 2.3 simulation tool: Visualization of the three-dimensional model of Compass gait bipedal walker (on the horizontal surface without disturbances) with the description of kinematic joints.

the dynamic multibody model and the selected control algorithms, an optimization problem for certain mechanical parameters as arguments, e.g. center of mass of the torso, can be formulated and eventually solved.

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