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DEFORMATION-DRIVEN FLOW IN HOMOGENIZED FLUID-SATURATED POROUS PERIODIC STRUCTURES: NONLINEARITY AND DYNAMIC EFFECTS

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Abstract: *The paper deals with the homogenization-based modelling of periodic porous structures saturated by a Newtonian fluid and locally controlable due to embedded piezoelectric segments which can induce a peristaltic deformation of the microchannels, in response to prescribed propagating voltage waves. To respect dynamic effects of the flow in bulged pores, where the advection term of the acceleration is important, two time scales are considered and an appropriate time scaling of the fast-slow dynamics is introduced in a proportion to the spatial scaling. The two-scale homogenized problem is nonlinear by the consequence of the advection term in the flow model. Further nonlinearity is introduced by deformation-dependent homogenized coefficients of the macroscopic equations. For this, a linear expansions based on the sensitivity analysis of the homogenized coefficients with respect to the deformation induced by the macroscopic quantities is employed.*

Keywords: Peristaltic deformation, Fluid flow, Homogenization, Porous media, Piezoelectric actuation.

1. Introduction

The peristaltic flow is induced by deforming wall of a channel. The study of this phenomenon is of a great importance in physiology, biomechanics, and, as a driving mechanism of fluid transport, also in the design of smart "bio-inspired" materials. We consider locally periodic porous structures occupying domain $\Omega = \Omega_s^\varepsilon \cup \Omega_f^\varepsilon \cup \Gamma_{fs}^\varepsilon$, constituted by the solid phase Ω_s^ε and the pores saturated by a Newtonian fluid flowing in channels $\Omega_f^\varepsilon = \Omega \setminus \overline{\Omega_s^\varepsilon}$, whereby ε is the microstructure scale. The solid phase $\Omega_s^\varepsilon = \Omega_m^\varepsilon \cup \Omega_*^\varepsilon \cup \Gamma_{m*}^\varepsilon$, where $\Omega_m^\varepsilon = \Omega_e^\varepsilon \cup \Omega_z^\varepsilon \cup \Gamma_{ez}^\varepsilon$, contains an elastic-dielectric part Ω_e^ε, and piezoelectric segments Ω_z^ε which can induce peristaltic deformation wave of the microchannels in response to the locally controlled electric field due to distributed electrodes Ω_*^ε connected to an external circuit. Such a smart material can transform a propagating electric potential wave into a peristaltic deformation wave propelling the fluid.

Using the asymptotic analysis w.r.t. the scale $\varepsilon \to 0$, pursuing our previous work (Rohan and Lukeš, 2018), we have developed a nonlinear homogenized model of such a smart porous material which describes the peristaltic-driven flow under a quasistatic regime, so disregarding any inertia effects (Rohan and Lukeš, 2021). Although the deformations are small and the channels do not collapse (the channel lumen is not closing completely by the deformation), the homogenized model can mimic the peristalsis-driven flow provided nonlinearity arising due to deformed configuration is accounted for conveniently. For this, we employed an approximation which leads to a nonlinear model as the consequence of the effective model parameters (tensors) depending on the solution by virtue of the homogenization respecting locally deforming microstructures (Rohan and Lukeš, 2015).

This paper presents some important issues to cope with when extending the homogenization problem for the dynamic flow, namely respecting the inertia in the fluid. First we introduce the micromodel which is subject of the asymptotic homogenization using the unfolding method (Cioranescu et. al, 2008), whereby a relevant scaling by ε of some micromodel parameters must be introduced. Then two different situations of respecting the flow dynamics are discussed.

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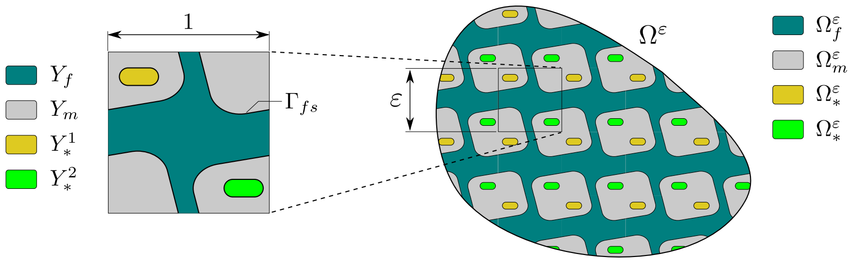

Fig. 1: Periodic microstructures and the representative cell Y decomposition.

2. Micromodel and the fluid-structure interaction problem

The periodic microstructure is generated by a rescaled reference periodic cell $Y =]0,1[^3$ decomposed into non-overlapping subdomains Y_m, Y_f and Y_*, see Fig. 1, $Y = Y_m \cup Y_f \cup Y_*$, such that $Y_m = Y_e \cup Y_z$, where the subdomains Y_d generate corresponding domains Ω_d^ε, $d = e, z, *, f$. For the device functionality, at least two groups of separated electrodes $\Omega_*^{\alpha,\varepsilon}$, $\alpha = 1, 2$ represented by $Y_*^\alpha \subset Y_*$ must be considered; domains $\Omega_*^{\alpha,\varepsilon} \subset \Omega_*^\varepsilon$ are constituted by separated inclusions allowing for the local voltage control. In the piezoelectric solid, the Cauchy stress tensor $\boldsymbol{\sigma}^\varepsilon$ and the electric displacement $\vec{D}^\varepsilon$ depend on the strain tensor $\boldsymbol{e}(\boldsymbol{u}^\varepsilon) = (\nabla \boldsymbol{u}^\varepsilon + (\nabla \boldsymbol{u}^\varepsilon)^T)/2$ defined in terms of the displacement field $\boldsymbol{u}^\varepsilon = (u_i^\varepsilon)$, and on the electric field $\vec{E}(\varphi^\varepsilon) = \nabla \varphi^\varepsilon$ defined in terms of the electric potential φ^ε. The following constitutive equations characterize the piezoelectric solid in Ω_s^ε and, thereby, for vanishing $\underline{\boldsymbol{g}}^\varepsilon$, also the elastic parts in Ω_e^ε and Ω_*^ε (infinite $\boldsymbol{d}^\varepsilon$),

$$\sigma_{ij}^\varepsilon(\boldsymbol{u}^\varepsilon, \varphi^\varepsilon) = A_{ijkl}^\varepsilon e_{kl}^\varepsilon(\boldsymbol{u}^\varepsilon) - g_{kij}^\varepsilon E_k^\varepsilon(\varphi^\varepsilon) , \quad D_k^\varepsilon(\boldsymbol{u}^\varepsilon, \varphi^\varepsilon) = g_{kij}^\varepsilon e_{ij}^\varepsilon(\boldsymbol{u}^\varepsilon) + d_{kl}^\varepsilon E_l^\varepsilon(\varphi^\varepsilon) , \tag{1}$$

where $\mathbb{A}^\varepsilon = (A_{ijkl}^\varepsilon)$ is the 4th-order elasticity symmetric positive definite tensor satisfying $A_{ijkl} = A_{klij} = A_{jilk}$, the deformation is coupled with the electric field through the 3rd order tensor $\underline{\boldsymbol{g}}^\varepsilon = (g_{kij}^\varepsilon)$, $g_{kij}^\varepsilon = g_{kji}^\varepsilon$ and $\boldsymbol{d} = (d_{kl})$ is the electric permittivity tensor. A Newtonian barotropic fluid is characterized by the dynamic viscosity μ^ε, a reference density ρ_f and the fluid compressibility γ, the stress tensor is

$$\boldsymbol{\sigma}_f^\varepsilon = -p^\varepsilon \boldsymbol{I} + 2\mu^\varepsilon(\mathbb{I} - \frac{1}{3}\boldsymbol{I} \otimes \boldsymbol{I})\boldsymbol{e}(\boldsymbol{v}^{f,\varepsilon}) . \tag{2}$$

The micromodel involves the following differential equations governing the fluid-solid interaction and the electric field coupled with the deformation through the piezoelectric constitutive law (1),

$$\begin{aligned} \rho_s \ddot{\boldsymbol{u}}^\varepsilon - \nabla \cdot \boldsymbol{\sigma}_s^\varepsilon(\boldsymbol{u}^\varepsilon, \varphi^\varepsilon) &= \boldsymbol{f}^{s,\varepsilon} , \quad \text{in } \Omega_s^\varepsilon , \\ -\nabla \cdot \vec{D}^\varepsilon(\boldsymbol{u}^\varepsilon, \varphi^\varepsilon) &= 0 , \quad \text{in } \Omega_z^\varepsilon \cup \Omega_e^\varepsilon , \\ \rho_f \left(\dot{\boldsymbol{v}}^{f,\varepsilon} + (\boldsymbol{w}^\varepsilon \cdot \nabla)\boldsymbol{v}^{f,\varepsilon} \right) + \nabla p^\varepsilon - \mu^\varepsilon \nabla^2 \boldsymbol{v}^{f,\varepsilon} &= \boldsymbol{f}^{f,\varepsilon} , \\ \gamma \dot{p} + \nabla \cdot \boldsymbol{v}^{f,\varepsilon} &= 0 , \quad \text{in } \Omega_f^\varepsilon , \end{aligned} \tag{3}$$

where $\boldsymbol{f}^{d,\varepsilon}$, $d = s, f$ are the body forces. Note that the "dot" means the material derivative and the seepage velocity $\boldsymbol{w}^\varepsilon = \boldsymbol{v}^{f,\varepsilon} - \dot{\tilde{\boldsymbol{u}}}^\varepsilon$ is defined using an extension $\dot{\tilde{\boldsymbol{u}}}^\varepsilon$ of the solid phase velocity.

3. Homogenization and the problem nonlinearity treatment

To retain specific features of the micromodel (3) in the homogenization limit $\varepsilon \to 0$, some of the involved material parameters are considered to depend on ε. To allow for microstructures with strongly controlled electric field, in formulation (3), spatially constant potentials $\bar{\varphi}^\alpha$ are given for each simply connected domain $\Omega_*^{\alpha,\varepsilon}$ occupied by the perfect conductor and represented by Y_*^α within the cell Y. The following assumptions are made where, for simplicity, parameters $\bar{\underline{\boldsymbol{g}}}, \bar{\boldsymbol{d}}$ and $\bar{\mu}$ are taken as constants:

$$\begin{aligned} &\text{a) Strongly controlled field:} \quad \varphi^{\alpha,\varepsilon} = \bar{\varphi}^\alpha \text{ in } \Omega_*^{\alpha,\varepsilon} , \\ &\text{b) Weakly piezoelectric material: } \underline{\boldsymbol{g}}^\varepsilon(x) = \varepsilon \bar{\underline{\boldsymbol{g}}} , \quad \boldsymbol{d}^\varepsilon(x) = \varepsilon^2 \bar{\boldsymbol{d}} , \quad \text{in } \Omega_z^\varepsilon \cup \Omega_e^\varepsilon , \\ &\text{c) viscous flow with the nonslip condition on pore walls,} \quad \mu^\varepsilon = \varepsilon^\beta \bar{\mu} , \quad \text{in } \Omega_f^\varepsilon , \end{aligned} \tag{4}$$

where $\beta = 2$, or $\beta = 3/2$, depending on the flow dynamics approximation. Above, the scaling b) is the consequence of the locally controlled electric field, a).

3.1. Nonsteady Stokes flow, neglected advection inertia

Neglecting the advection term $(\boldsymbol{w}^\varepsilon \cdot \nabla)\boldsymbol{v}^{f,\varepsilon}$ in (3) leads to the same scaling, as in the quasi-static case, $\beta = 2$ in (4)(c). The homogenization using the approach explained in (Rohan and Naili, 2020) provides an extension of the piezo-poroelastic model derived in (Rohan and Lukeš, 2018) for disconnected porosity. The macroscopic displacement $\boldsymbol{u}^0$, the flow seepage $\boldsymbol{w}^0$ and pressure p^0 satisfy the following coupled equations involving the homogenized (effective) coefficients $\mathbb{H} := (\mathbb{A}, \boldsymbol{B}, M, Z^\alpha, \boldsymbol{H}^\alpha)$ of the piezo-poroelastic model, and the dynamic permeability $\boldsymbol{K}(t)$, such that

$$\bar{\rho}\partial_{tt}^2 \boldsymbol{u}^0 + \phi_f \rho_f \partial_t \boldsymbol{w} - \nabla \cdot \left(\mathbb{A}\boldsymbol{e}(\boldsymbol{u}) - p^0 \boldsymbol{B} + \sum_\alpha \boldsymbol{H}^\alpha \bar{\varphi}^\alpha \right) = \hat{\boldsymbol{f}} \,,$$

$$\boldsymbol{B} : \boldsymbol{e}(\partial_t \boldsymbol{u}^0) + M \partial_t p^0 + \nabla \cdot \boldsymbol{w} = \sum_\alpha Z^\alpha \partial_t \bar{\varphi}^\alpha \,, \quad (5)$$

$$\text{where} \quad \boldsymbol{w}(t,x) = -\frac{1}{\bar{\mu}} \int_0^t \boldsymbol{K}(t-s)(\nabla p(s,\cdot) - \boldsymbol{f}^f(s,\cdot))\,\mathrm{d}t \,,$$

where $\bar{\rho}$ and $\phi_f \rho_f$ are the effective densities of the mixture and of the fluid, respectively. For a suitable given electric potential control $\{\bar{\varphi}^\alpha\}_\alpha(t,x)$, a peristaltic deformation wave propagates. However, model (5) can induce a fluid flow opposed to the one caused by the pressure gradient only when the nonlinearity associated with respecting the deformed configuration effect of the homogenized coefficients is captured. Using the sensitivity analysis approach explained in (Rohan and Lukeš, 2015), all the coefficients $\mathbb{H}$ in (5) can be replaced by $\tilde{\mathbb{H}}$ introduced using the first order expansion formulae which have the generic form

$$\tilde{\mathbb{H}}(\boldsymbol{e}(\boldsymbol{u}), p) = \mathbb{H}^0 + \delta_{\boldsymbol{e}} \mathbb{H}^0 : \boldsymbol{e}(\boldsymbol{u}) + \delta_p \mathbb{H}^0 p + \sum_\alpha \delta_{\varphi,\alpha} \mathbb{H}^0 \bar{\varphi}^\alpha \,, \quad (6)$$

where $\delta_\spadesuit \mathbb{H}^0$ is the gradient of $\mathbb{H}^0$ w.r.t. the macroscopic quantity $\spadesuit$. In the case of the permeability $\boldsymbol{K}(t)$, an expansion analogous to (6) based on the sensitivity analysis is not obvious. A possible treatment is based on the spectral decomposition of the seepage $\boldsymbol{w}$ using the eigenfunctions $\{\boldsymbol{\omega}^r\}_r$ and eigenvalues $\{\lambda^r\}_r$ of the eigenvalue problem Find $(\boldsymbol{\omega}^r, \pi^r) \in \mathbf{H}^1_{\#0}(Y_f) \times H^1_\#(Y_f)$, such that

$$a_f(\boldsymbol{\omega}^r, \boldsymbol{\psi}) + \langle \nabla_y \pi^r, \boldsymbol{\psi} \rangle_{Y_f} = \lambda^r \langle \boldsymbol{\omega}^r, \boldsymbol{\psi} \rangle_{Y_f} \,, \quad \langle \nabla_y q, \boldsymbol{\omega}^r \rangle_{Y_f} = 0 \,, \quad \langle \boldsymbol{\omega}^r, \boldsymbol{\omega}^r \rangle_{Y_f} = 1 \,, \quad (7)$$

for all $(\boldsymbol{\psi}, q) \in \mathbf{H}^1_{\#0}(Y_f) \times H^1_\#(Y_f)$; by $\#$ we label a space of Y-periodic functions. Then the seepage is expressed using the basis $\{\boldsymbol{\omega}^r\}_r$, so that $\boldsymbol{w}(t,x,y) = \sum_r \boldsymbol{\omega}^r(y)\vartheta^r(t,x)$. Respecting the deformation of the microconfiguration due to the response at time t makes the eigenpairs $(\boldsymbol{\omega}^r_{(t)}, \lambda^r_{(t)})$ to depend on t, as indicated by $_{(t)}$. Upon denoting by $\langle v \rangle_{Y_f} = |Y|^{-1} \int_{Y_f} v$ the average, the seepage is given by

$$\boldsymbol{w}(t,x) = -\int_0^t \boldsymbol{\mathcal{K}}(t,s) \nabla p^0(s,x)\,\mathrm{d}s - \sum_r \left\langle \boldsymbol{\omega}^r_{(t)} \right\rangle_{Y_f} e^{-\lambda^r_{(t)} t} \int_0^t e^{\lambda^r_{(s)} s} \left\langle \boldsymbol{\omega}^r_{(s)} \right\rangle_{Y_f} \cdot \nabla p^0(s,x)\,\mathrm{d}s \,, \quad (8)$$

where the force $\boldsymbol{f}^f$ is disregarded for the brevity. Hence, a nonlinear expression of the permeability, the convolution kernel $\boldsymbol{\mathcal{K}}(t,s)$ is obtained, since $(\boldsymbol{\omega}^r_{(t)}, \lambda^r_{(t)})$ depend on the micro-deformation of channels Y_f.

3.2. Navier-Stokes flow model, limit model with two-scales in space and time

In the case of the steady flow, the homogenization leads to the Forchheimer law, see *e.g.* (Peszyńska and Trykozko, 2010; Rohan and Naili, 2020), in the nonsteady case, the treatment is even more delicate, cf. . (Mikelic, 1991). To retain both the parts of the inertia $\rho_f(\dot{\boldsymbol{v}}^{f,\varepsilon} + (\boldsymbol{w}^\varepsilon \cdot \nabla)\boldsymbol{v}^{f,\varepsilon}$ in the limit $\varepsilon \to 0$ a modified scaling of the viscosity is needed; we consider $\beta = 3/2$ and the asymptotic expansion of the unfolded $\mathcal{T}_\varepsilon(\boldsymbol{w}^\varepsilon(t,x)) = \varepsilon^{1/2}(\boldsymbol{w}^0(t,\tau,x,y) + \varepsilon \boldsymbol{w}^1(t,\tau,x,y)\dots)$ where $\tau = \varepsilon^{-1/2} t$ is the "fast time"; recall $y = x/\varepsilon$

is the "fast spatial coordinate". Using asymptotic expansions for $\boldsymbol{u}^\varepsilon$, φ^ε and p^ε, as in the quasistatic case, the following limit equations governing the flow can be obtained,

$$\begin{aligned} \rho_f(\partial_\tau + \boldsymbol{w}^0 \cdot \nabla_y)(\boldsymbol{w}^0 + \partial_\tau \widetilde{\boldsymbol{u}}^1) + \nabla_x p^0 + \nabla_y p^1 - \bar{\mu}\nabla_y^2(\boldsymbol{w}^0 + \partial_\tau \widetilde{\boldsymbol{u}}^1) = 0 \;, \\ \gamma \partial_\tau p^0 + \nabla_y \cdot (\boldsymbol{w}^0 + \partial_\tau \widetilde{\boldsymbol{u}}^1) + \nabla_x \cdot \partial_\tau \boldsymbol{u}^0 = 0 \;, \end{aligned} \tag{9}$$

$$\begin{aligned} \gamma \partial_t p^0 + \nabla_x \cdot \partial_t \boldsymbol{u}^0 + \nabla_y \cdot \partial_t \widetilde{\boldsymbol{u}}^1 = 0 \;, \\ \gamma \partial_\tau p^1 + \nabla_x \cdot (\boldsymbol{w}^0 + \partial_\tau \widetilde{\boldsymbol{u}}^1) = 0 \;, \end{aligned} \tag{10}$$

where all two-scale time-space functions are Y, T-periodic in (y, τ). The two-scale limit equations governing the solid deformations involve the two-scale acceleration $\partial_{tt}^2 \boldsymbol{u}^0 + \partial_{\tau\tau}^2 \boldsymbol{u}^1$, but otherwise take the same form, as in the quasistatic case. Without specifying mathematical details, namely the functional spaces, we present the weak form with the test functions $\boldsymbol{v}^0(x)$, $\boldsymbol{v}^1(x, y)$, and $\hat{\psi}^0(x, y)$,

$$\begin{aligned} &\int_\Omega \fint_{Y_s} \rho_s(\partial_{tt}^2 \boldsymbol{u}^0 + \partial_{\tau\tau}^2 \boldsymbol{u}^1) \cdot \boldsymbol{v}^0 \\ &+ \int_\Omega \fint_{Y_s} \left(\mathbb{A} \left(\boldsymbol{e}_x(\boldsymbol{u}^0) + \boldsymbol{e}_y(\boldsymbol{u}^1) \right) - \underline{\bar{\boldsymbol{g}}}^T \nabla_y \hat{\varphi}^0 \right) : \left(\boldsymbol{e}_x(\boldsymbol{v}^0) + \boldsymbol{e}_y(\boldsymbol{v}^1) \right) = \int_\Omega p^0 \fint_{\Gamma_f} \boldsymbol{v}^1 \cdot \boldsymbol{n}^f \;, \\ &\int_\Omega \fint_{Y_s} \nabla_y \hat{\psi}^0 \cdot \left[\underline{\bar{\boldsymbol{g}}} : \left(\boldsymbol{e}_x(\boldsymbol{u}^0) + \boldsymbol{e}_y(\boldsymbol{u}^1) \right) + \bar{\boldsymbol{d}} \nabla_y \hat{\varphi}^0 \right] = 0 \;. \end{aligned} \tag{11}$$

Note that $\boldsymbol{w}^0$ is involved in $(9)_1$ in a nonlinear form. Assuming a T-periodicity w.r.t. the time τ describing a fast dynamics, the system (9), (10) and (11) simplifies upon time averaging in τ. Further details will be reported in a forthcoming publication.

4. Conclusion

A model of electroactive porous material has been derived using the homogenization of the linearized fluid-structure interaction problem. As confirmed by numerical studies, to achieve the desired pumping effect of the homogenized continuum, it is necessary to account for the nonlinearity associated with deformation-dependent microconfigurations and, hence, respecting deformation-dependent effective properties of the homogenized material. For this, the sensitivity analysis approach has been applied which leads to a computationally efficient numerical scheme for solving the nonlinear problem. As a new contribution, dynamic aspects of the peristaltic deformation driven flow has been studied in the context of the homogenization. Two different treatments of the flow inertia have been examined, leading to different homogenized models.

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References

Cioranescu, D., Damlamian, A., Griso, G. (2008), The periodic unfolding method in homogenization. *Siam J. Math. Anal.*, 40(4):1585–1620.

Mikelic, A., 1991. Homogenization of nonstationary Navier-Stokes equations in a domain with a grained boundary. Annali di Matematica Pura ed Applicata 158, 167–179.

Peszyńska, M., Trykozko, A. (2010), Forchheimer law in computational and experimental studies of flow through porous media at porescale and mesoscale. In: *Current Advances in Nonlinear Analysis and Related Topics.* Vol. 32 of GAKUTO International Series. pp. 463–482.

Rohan, E., Lukeš, V. (2015), Modeling nonlinear phenomena in deforming fluid-saturated porous media using homogenization and sensitivity analysis concepts. *Applied Mathematics and Computation*, Vol. 267, pp 583–595.

Rohan, E., Lukeš, V. (2018), Homogenization of the fluid-saturated piezoelectric porous media. *International Journal of Solids and Structures 147*, Vo. 167, pp 110–125.

Rohan, E. Naili, S. (2020), Homogenization of the fluid-structure interaction in acoustics of porous media perfused by viscous fluid. *Z. Angew. Math. Phys.*, 71(137).

Rohan, E., Lukeš, V. (2021), Multiscale modelling of flow due to the peristaltic wave in deforming poro-piezoelectric medium. In: *Proc of the IX Int. Conf. on Computational Meth. for Coupled Problems in Sci. and Engrg., COUPLED PROBLEMS 2021*, DOI: 10.23967/coupled.2021.037